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Chapter P

Preliminary Concepts

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To introduce

- The Real Number System
- Integer and Rational Numbr Exponents
- Polynomials
- Factoring
- Rational Expressions
- Complex Numbers

P.1 The Real Number System

Objectives

To introduce

- Set of Numbers.
- Union and Intersection of Sets.
- Absolute Value and Distance.
- Interval Notation.
- Order of Operations.
- Simplify Variable Expressions.

I. **True or False Statements (Give a reason):**

1. The sum of two irrational numbers is an irrational number.

   Wrong Answer. True. Reason $\sqrt{3}-\sqrt{2}$ is an irrational number.

   Correct Answer. False. $\sqrt{3}-\sqrt{3}=0$ which is not irrational.

2. The set of irrational numbers is closed under multiplication.

   Wrong Answer. True. For example, $\sqrt{2}\cdot\sqrt{3}=\sqrt{6}$ which is an irrational number.

   Correct Answer. False. For example, $\sqrt{2}\cdot\sqrt{2}=2$ is not irrational.

3. The number 0 is both rational and irrational.

   Wrong Answer. True. 0 can be considered as rational and irrational.

   Correct Answer. False. An irrational number cannot be written in the form

   $$\frac{\text{integer}}{\text{nonzero integer}}$$

   and since $0=\frac{0}{1}$, therefore 0 is a rational number and not an irrational number.

4. The number 1.414 is an irrational number.

Wrong Answer. True, since $1.414 = \sqrt{2}$.

Correct Answer. False, since $1.414 = \frac{1414}{1000}$, which is a rational number.

5. The number π is a rational number since $\pi = \frac{22}{7}$.

Wrong Answer. True, since $\frac{22}{7}$ is a rational number.

Correct Answer. False. $\frac{22}{7}$ is an approximation of π and not equal to π.

6. $0.\overline{32}$ is a rational number because $0.\overline{32} = \frac{32}{100}$.

Wrong Answer. True. Because $\frac{32}{100}$ is a rational number.

Correct Answer. False. $0.\overline{32}$ is a rational number since it is in a repeating decimal form $0.\overline{32} = 0.323232\cdots$.

7. The set of nonzero integers is closed under division.

Wrong Answer. True. For exmaple, $\frac{10}{2} = 5$.

Correct Answer. False. $\frac{3}{2}$ is not an integer.

8. Every real number has a multiplicative inverse.

Wrong Answer. True. $\frac{1}{x}$ is the multiplicative inverse of x.

Correct Answer. False. The number 0 has no multiplicative invese since $\frac{1}{0}$ is undefined.

9. Every odd number is a prime number.

Wrong Answer. True. 3 is both odd and prime.

Correct Answer. False. 9 is odd and not prime.

10. Every composite number is NOT an odd number.

Wrong Answer. True. 10 is a composite number which is not odd.

Correct Answer. False. 9 is both an odd and a composite number.

11. Let $A = \{x | x \text{ is a composite}, \quad 1 \leq x \leq 10\}$ and $B = \{x | x \text{ is a prime}, \quad 1 \leq x \leq 10\}$, then $A \cap B = \{2\}$.

    Wrong Answer. True. 2 is the only number which is both composite and prime.

    Correct Answer. False. $A = \{4, 6, 8, 9, 10\}$ while $B = \{2, 3, 5, 7\} \Rightarrow A \cap B = \phi$.

12. Let $A = \left\{-2, \frac{3}{2}, 5, 11\right\}$ and $B = \{-11, -6, 0, 5, 11\}$ then $A \cup (A \cap B) = A \cap B$.

    Wrong Answer. True. $A \cup (A \cap B) = (A \cup A) \cap B = A \cap B$

    Correct Answer. False. $A \cap B = \{5, 11\} \subset A \Rightarrow A \cup (A \cap B) = A$.

13. If x is any real number, then $|-x| = x$.

    Wrong Answer. True. $|-x| = |-1|x = x$.

    Correct Answer. False. If $x = -2 \Rightarrow |-(-2)| = 2 \neq -2$.

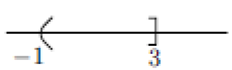
14. On a real number line, the distance between the numbers x and -5 is $|x - 5|$.

    Wrong Answer. True.

    Correct Answer. False. The distance $= |x - (-5)| = |x + 5|$.

15. $\{x | -1 < x \leq 3\} \cap \{x | 1 < x < 5\} = \{x | 1 \leq x \leq 3\}$.

    Wrong Answer. True. The graph of $\{x | -1 < x \leq 3\}$ is



    and the graph of $\{x | 1 < x < 5\}$ is

Thus from the graphs, the intersection is $[1,3]$.

Correct Answer. False. From the graphs, 1 is not in the given intersection $\Rightarrow$ The intersection $= (1,3]$.

II. **Solutions With Wrong Steps**:

(A) The Distributive Property:

1. $-3(x-y) =$

   Wrong Solution: $-3x - 3y$.

   Correct Solution: $(-3)x + (-3)(-y) = -3x + 3y$.

2. $(-3)(x+2y) =$

   Wrong Solution: $(-3)x - 2y = -3x - 2y$

   Correct Solution: $(-3)x + (-3)(2y) = -3x - 6y$.

3. $-3[(3x-y) - (x+5y)] =$

   Wrong Solution: $-3[3x - y - x + 5y] = -3[2x + 4y]$

   $$= \left\langle \begin{array}{l} -6x + 12y \\ \text{OR} \\ -6x + 4y \end{array} \right. .$$

   Correct Solution: $-3[3x - y - x - 5y] = -3[2x - 6y] = -6x + 18y$.

4. Simplify $3 + 4(2x + 3y) =$

   Wrong Solution: $7(2x + 3y) = 14x + 21x$

   Correct Solution: $3 + (8x + 12y) = 3 + 8x + 12y$.

(B) The Commutative Property for Addition: The set of real numbers is commutative under subtraction.

Wrong Solution: True. $x - y = -y + x$.

Correct Solution: False. $x - y \neq y - x$.

(C) Associative Property for Addition: The set of real numbers is associative under subtraction.

Wrong Solution: True, since $x - (y - z) = (x - y) - z$.

Correct Solution: False. Since $x - (y - z) = x - y + z \neq (x - y) - z = x - y - z$.

(D) Simplify each of the following:

1. $-3 \cdot 5^2 =$

   Wrong Solution: $((-3)(5))^2 = (-15)^2 = 225$.

   Correct Solution: $(-3)(25) = -75$.

2. $(-2x)^5 =$

   Wrong Solution: $-2x^5$.

   Correct Solution: $(-2)^5(x)^5 = -32x^5$.

(E) Evaluate $xy - z(x - y)^2$ for $x = 3, \quad y = -2$ and $z = -1$.

Wrong Solution: $(3)(-2) - (-1)(3 - (-2))^2 = -6 - (-3 - 2)^2 = -6 - (25) = -31$.

Correct Solution: $(3)(-2) - (-1)(3 - (-2))^2 = -6 + (5)^2 = -6 + 25 = 19$.

P.2 Integer and Rational Number Exponents

Objectives

To introduce

- Properties of Exponents
- Scientific Notation
- Rational Exponents and Radicals
- Simpify Radical Expressions

I. **True or False Statements (Give a reason):**

1. $(-8)^0 = -1$.

   Wrong Answer. True, since $(-8)^0 = -8^0 = -1$.

   Correct Answer. False. Since $x^0 = 1$ for any nonzero real number. Therefore, $(-8)^0 = 1$.

2. $(-2)^3(-3)^2 = 72$.

   Wrong Answer. True, since $(-2)^3 \cdot (-3)^2 = 2^3 \cdot 3^2 = 72$.

   Correct Answer. False. Since $(-2)^3(-3)^2 = (-8)(9) = -72$.

3. $0^{-1} = 0$.

   Wrong Answer. True, since $0^n = 0$ for any integer n.

   Correct Answer. False. 0^{-1} is an undefined expression, because $\frac{1}{x}$ is undefined for $x = 0$.

4. $0^p = 0$ for any rational number p.

   Wrong Answer. True. $0^p = 0$ for any rational number p.

   Correct Answer. False. Since 0^p is undefined for $p < 0$, while $0^p = 0$ for $p > 0$.

5. The expression $\dfrac{a^0 b^{-8}}{3c-5}$ is defined for any real numbers a, b, and c, where $c \neq \dfrac{5}{3}$.

Wrong Answer. True.

Correct Answer. False. It is true only for $a \neq 0$, $b \neq 0$, and $c \neq \dfrac{5}{3}$.

6. $x^4 y^2 = (xy)^6$.

Wrong Answer. True, since we can add powers.

Correct Answer. False. Powers can not be added since the bases are not the same.

7. $\left(\dfrac{x}{5y}\right)^{-3} = \dfrac{1}{5x^3 y}$.

Wrong Answer. True. The expression $\left(\dfrac{x}{5y}\right)^{-3} = \dfrac{x^{-3}}{5y} = \dfrac{1}{5x^3 y}$.

Correct Answer. False. The expression $\left(\dfrac{x}{5y}\right)^{-3} = \dfrac{x^{-3}}{5^{-3} y^{-3}} = \dfrac{125 y^3}{x^3}$.

8. The number $\dfrac{2.5 \times 10^8}{5 \times 10^{10}}$ in scientific notation is 0.5×10^{-2}.

Wrong Answer. True. $\dfrac{2.5 \times 10^8}{5 \times 10^{10}} = \dfrac{2.5}{5} \times \dfrac{10^8}{10^{10}} = 0.5 \times 10^{-2}$.

Correct Answer. False. It must be 5×10^{-3}.

A number written in scientific notation has the form $a \times 10^n$ where n is an integer and $1 \leq a < 10$.

II. **Solutions with Wrong Steps**:

Simplify each of the following expressions:

1. $\left(\dfrac{16x^2}{y^4}\right)^{3/2}$ where x and y are any real numbers.

Wrong Solution $= \dfrac{(4^2)^{3/2}(x^2)^{3/2}}{(y^4)^{3/2}} = \dfrac{4^3 x^3}{y^6} = \dfrac{64x^3}{y^6}$.

Correct Solution $= \dfrac{(4^2)^{3/2}(x^2)^{3/2}}{(y^4)^{3/2}} = \dfrac{4^3 |x^3|}{|y^6|} = \dfrac{64x^2|x|}{y^6}$.

Notice that $\sqrt{A^2} = |A|$ where A is any real number.

2. $\sqrt{x^2 + 2xy + y^2}$ where $x < 0$ and $y < 0$.

   Wrong Solution $= \sqrt{(x+y)^2} = (x+y)$.

   Correct Solution $= \sqrt{(x+y)^2} = |x+y| = -(x+y)$ because $x + y < 0$.

3. $\sqrt{\sqrt[3]{x}}$ for $x > 0$.

   Wrong Solution $= \sqrt[5]{x}$.

   Correct Solution $= \left(x^{1/3}\right)^{1/2} = x^{1/6} = \sqrt[6]{x}$.

4. $\dfrac{x^4}{x^{-2}}$.

   Wrong Solution $= x^4 \cdot x^2 = x^8$.

   Correct Solution $= x^4 \cdot x^2 = x^{4+2} = x^6$.

5. $\dfrac{x^4 y^{-3/2}}{x^{-2} y^{11/2}}$.

   Wrong Solution $= \dfrac{x^4 + x^2}{y^{11/2} + y^{3/2}}$.

   Correct Solution $= \dfrac{x^4 \cdot x^2}{y^{11/2} y^{3/.2}} = \dfrac{x^6}{y^7}$.

6. $\sqrt{\sqrt[3]{y^{-3/2}}}, \quad y > 0$.

   Wrong Solution $= \sqrt{y^{-1/2}} = y^{-1} = \dfrac{1}{y}$.

   Correct Solution $= \sqrt{y^{-1/2}} = y^{-1/4} = \dfrac{1}{\sqrt[4]{y}} = \dfrac{\sqrt[4]{y^3}}{y}$.

7. $(\sqrt{4x^2 + 25} - 3)(\sqrt{4x^2 + 25} + 3)$.

   Wrong Solution $= 2x + 5 - 9 = 2x - 4$.

   Correct Solution $= (4x^2 + 25) - 9 = 4x^2 + 16$.

8. $\left[x^{-2m+4}\right]^{-3/2}, \quad x > 0$.

Wrong Solution $= x^{(-3/2)(-2m)+4} = x^{3m+4}$

Correct Solution $= x^{(-3/2)(-2m+4)} = x^{3m-6}$.

9. $3m^2n\sqrt[3]{n} - 16m^2n\sqrt[3]{n}$.

   Wrong Solution $= -13m^2n - 13\sqrt[3]{n}$.

   Correct Solution $= (3m^2n - 16m^2n)\sqrt[3]{n} = -13m^2n\sqrt[3]{n}$.

10. $\sqrt[3]{8x^3y^4}$.

    Wrong Solution $= 8(x^3 \cdot y^3y)^{1/3} = 8xy\sqrt[3]{y}$.

    Correct Solution $= (2^3x^3y^3y)^{1/3} = 2xy\sqrt[3]{y}$.

11. $\sqrt{\sqrt[3]{m^{54}}}, \quad m > 0$.

    Wrong Solution $= \left(m^{54}\right)^{3/2} = \left(m^{27}\right)^3 = m^{81}$.

    Correct Solution $= \left(m^{54}\right)^{1/6} = m^9$.

12. $\sqrt[3]{y^{-4}}$.

    Wrong Solution $y\sqrt[3]{y^{-1}} = \dfrac{y}{\sqrt[3]{y}} = \sqrt[3]{y^2}$.

    Correct Solution $\sqrt{y^{-3}y^{-1}} = y^{-1}\sqrt[3]{y^{-1}} = \dfrac{1}{y\sqrt[3]{y}} = \dfrac{\sqrt[3]{y^2}}{y^2}$.

13. $\dfrac{4x^3y^{-4}x^4y^6}{x^4}$.

    Wrong Solution $= \dfrac{(4x^7)(4y^2)}{x^4} = \dfrac{4x^7}{x^4} \cdot 4y^2 = 16x^3y^2$.

    Correct Solution $= 4\dfrac{x^7}{x^4} \cdot y^2 = 4x^3y^2$.

14. $5y\sqrt[3]{64y^4} - \sqrt[3]{125y^7}$.

    Wrong Solution $= 5y\sqrt[3]{64y^4 - 125y^7} = 5y(4y\sqrt[3]{y} - 5y^2\sqrt[3]{y}) = 20y^2\sqrt[3]{y} - 25y^3\sqrt[3]{y}$.

    Correct Solution $= 5y\sqrt[3]{4^3y^3y} - \sqrt[3]{5^3y^6y} = 5y(4y)\sqrt[3]{y} - 5y^2\sqrt[3]{y} = 20y^2\sqrt[3]{y} - 5y^2\sqrt[3]{y} = 15y^2\sqrt[3]{y}$.

III. Rationalize the Denominator:

1. $\dfrac{1}{\sqrt[3]{x}}$.

<u>Wrong Solution</u> $= \dfrac{\sqrt[3]{x}}{\sqrt[3]{x}\sqrt[3]{x}} = \dfrac{\sqrt[3]{x}}{x}$.

<u>Correct Solution</u> $= \dfrac{\sqrt[3]{x^2}}{\sqrt[3]{x}\sqrt[3]{x^2}} = \dfrac{\sqrt[3]{x^2}}{\sqrt[3]{x^3}} = \dfrac{\sqrt[3]{x^2}}{x}$.

2. $\sqrt{\dfrac{5}{x^2+y^2}}$.

<u>Wrong Solution</u> $= \dfrac{\sqrt{5}}{\sqrt{x^2+y^2}} = \dfrac{\sqrt{5}}{x+y}$.

<u>Correct Solution</u> $= \sqrt{\dfrac{5(x^2+y^2)}{(x^2+y^2)^2}} = \dfrac{\sqrt{5(x^2+y^2)}}{x^2+y^2}$.

3. $\dfrac{6}{\sqrt[3]{2}} - \dfrac{5}{4\sqrt[3]{16}}$.

<u>Wrong Solution</u> $= \dfrac{6\sqrt[3]{2}}{2} - \dfrac{5\sqrt[3]{16}}{4(16)} = 3\sqrt[3]{2} - \dfrac{(5)(2)\sqrt[3]{2}}{(4)(16)} = 3\sqrt[3]{2} - \dfrac{5}{32}\sqrt[3]{2} = \dfrac{91}{32}\sqrt[3]{2}$.

<u>Correct Solution</u> $\dfrac{6\sqrt[3]{4}}{\sqrt[3]{8}} - \dfrac{5}{8\sqrt[3]{2}} = \dfrac{6}{2}\sqrt[3]{4} - \dfrac{5}{8}\dfrac{\sqrt[3]{4}}{2} = 3\sqrt[3]{4} - \dfrac{5\sqrt[3]{4}}{16} = \dfrac{43}{16}\sqrt[3]{4}$.

P.3 Polynomials

Objectives

To introduce

- Polynomials
- Operations on Polynomials

I. **True or False Statements (Give a Reason):**

1. The leading coefficient of the polynomial $5x + 7 + 9x^3 + 3x^2$ is 5.

   Wrong Answer. True, since 5 is the coefficient of x in the first term.

   Correct Answer. False. The leading coeffichent is 9. Notice that the polynomial must be written in standard form first, i.e., $9x^3 + 3x^2 + 5x + 7$.

2. The expression $7x^{10} + 3x^4 + 2x + 6x^{-1} + 8$ is a polynomial in x of degree 10.

   Wrong Answer. True, since 10 is the largest power of x.

   Correct Answer. False. The expression is not a polynomial because of the term $6x^{-1}$ (all powers of x must be non-negative integers).

3. The degree of the polynomial $3x^8 - 4x^3y^3 + 5x^2y^7 - 3y$ is 8.

   Wrong Answer. True.

   Correct Answer. False. The degree is 9. Notice that in case of a polynomial in two or more variables, we add powers of the variables in the same term to determine the degree.

4. If P and Q are two polynomials of degree n, then $P + Q$ is also of degree n.

   Wrong Answer. True, since $P(x) = x^2$ and $Q(x) = 3x^2 + 1 \Rightarrow P(x) + Q(x) = 4x^2 + 1$ is of degree 2.

Correct Answer. False. Let $P(x) = x^2 + 3x + 1$ and $Q(x) = -x^2 + x - 3 \Rightarrow P(x) + Q(x) = 4x - 2$ which is of degree 1 and not of degree 2.

5. If P is a polynomial of degree 3, then P^2 is a polynomial of degree 9.

   Wrong Answer. True, since $(3)^2 = 9$.

   Correct Answer. False. If $P = x^3$, then $P^2 = (x^3)^2 = x^6$ which is of degree 6.

II. **Solutions with Wrong Steps**:

1. Find the coefficient of x^2y^2 in the expression $2y(2x - 3xy)^2$.

   Wrong Solution. The expression $= (4xy - 6xy^2)^2 = 16x^2y^2 - 48x^2y^3 + 36x^2y^4 \Rightarrow$ The coefficient of $x^2y^2 = 16$.

   Correct Solution. The expression $= 2y(4x^2 - 12x^2y + 9x^2y^2) = 8x^2y - 24x^2y^2 + 18x^2y^3 \Rightarrow$ The coefficient of $x^2y^2 = -24$.

2. Find the product $(\sqrt[3]{x} - \sqrt[3]{y})(\sqrt[3]{x} + \sqrt[3]{y})$.

   Wrong Solution. $= x - y$.

   Correct Solution. $= (\sqrt[3]{x})^2 - (\sqrt[3]{y})^2 = \sqrt[3]{x^2} - \sqrt[3]{y^2}$.

3. Find the product $\left(\frac{1}{2}x - \frac{1}{3}y\right)(2x + 3y)$.

   Wrong Solution. $= x^2 - y^2$.

   Correct Solution. $= \left(\frac{1}{2}x\right)(2x) + \left(\frac{3}{2} - \frac{2}{3}\right)xy - \left(\frac{1}{3}\right)(3)y^2 = x^2 + \frac{5}{6}xy - y^2$.

4. Find the product $(5y^m + 2)(3y^m - 4)$.

   Wrong Solution. $= 15y^{m^2} + (6 - 20)y^m - 8 = 15y^{m^2} - 14y^m - 8$.

   Correct Solution. $= 15(y^m)^2 - 14y^m - 8 = 15y^{2m} - 14y^m - 8$.

P.4 Factoring

Objectives

To introduce

- Greatest Common Factor (GCF)
- Factoring Trinomials
- Special Factoring
- Factor by Grouping
- General Factoring Strategy

I. True or False Statements (Give a Reason):

1. 2^8 is the GCF of $2^3, 2^5$ and 2^8.

   Wrong Answer. True. Reason 2^8 has the greatest exponent.

   Correct Answer. False, since 2^3 is the GCF.

2. $9ac$ is the GCF of $27a^3$ and $18ac$.

   Wrong Answer. True.

   Correct Answer. False. $9a$ is the GCF.

3. No GCF of $3(2x+5)$ and $4(2x+5)$.

   Wrong Answer. True.

   Correct Answer. False. $(2x+5)$ is the GCF.

4. $x^2 - 5x - 14 = (x-2)(x+7)$.

   Wrong Answer. True.

   Correct Answer. False, since $x^2 - 5x - 14 = (x+2)(x-7)$.

5. $x^2 + 8x + 15 = (x - 3)(x - 5)$.

Wrong Answer. True.

Correct Answer. False, since $x^2 + 8x + 15 = (x + 3)(x + 5)$.

6. $6x^2 - 11x + 4$ is non-factorable over the integers.

Wrong Answer. True, since there are no two numbers whose product is 4 and whose sum or difference is -11.

Correct Answer. False, since $6x^2 - 11x + 4 = (2x - 1)(3x - 4)$

7. $x^2 + 8x - 7 = (x + 7)(x + 1)$.

Wrong Answer. True.

Correct Answer. False, since $(x + 7)(x + 1) = x^2 + 8x + 7$ and not $x^2 + 8x - 7$.

8. $x^2 + x = x^3$.

Wrong Answer. True.

Correct Answer. False, since $x^2 + x = x(x + 1)$.

9. $x^2 - 5x = -4x^2$.

Wrong Answer. True.

Correct Answer. False, since $x^2 - 5x = x(x - 5)$.

10. $x^2 - 12 = (x - 3)(x + 4)$.

Wrong Answer. True.

Correct Answer. False, since $(x - 3)(x + 4) = x^2 + x - 12$.

11. $(x^{3n} + y^{6m}) = (x^n + y^{2m})^3$.

Wrong Answer. True, since $(a + b)^3 = a^3 + b^3$.

Correct Answer. False, since $(x^n + y^{2m})^3 = x^{3n} + 3x^{2n}\, y^{2m} + 3x^n\, y^{4m} + y^{6m}$.

12. $a^2 + ba = a(ba)$.

    Wrong Answer. True.

    Correct Answer. False, since $a^2 + ba = a(a + b)$.

13. $x^2 + 4 = (x + 2)(x - 2)$.

    Wrong Answer. True.

    Correct Answer. False, since $(x + 2)(x - 2) = x^2 - 4 \neq (x^2 + 4)$.

II. Solutions with Wrong Steps:

Factor completely:

1. $10x^3 + 6x =$

   Wrong Solution. $x(10x^2 + 6)$.

   Correct Solution. $2x(5x^2 + 3)$.

2. $10x^3 + 6x =$

   Wrong Solution. $2x(5x^2 + 6)$.

   Correct Solution. $2x(5x^2 + 3)$.

3. $10x^3 + 6x =$

   Wrong Solution. $2x(5x^2 + 3x) = 2x^2(5x + 3)$.

   Correct Solution. $2x(5x^2 + 3)$.

4. $15x^{2n} + 9x^{n+1} - 3x^n =$

   Wrong Solution. $3x^n(5x^2 + 3x - 1)$.

   Correct Solution. $3x^n(5x^n + 3x - 1)$.

5. $(m+5)(x+3)+(m+5)(x-10)=$

<u>Wrong Solution.</u> $(m+5)(x+3)(x-10)$

<u>Correct Solution</u>. $(m+5)((x+3)+(x-10))=(m+5)(2x-7)$.

6. $x^2-5x-14=$

<u>Wrong Solution.</u> $x(x-5)-14$.

<u>Correct Solution</u>. $(x+2)(x-7)$.

7. $x^4+5x^2+6=$

<u>Wrong Solution.</u> $x^4+5x^2+6=(x^2)^2+5(x^2)+6$. Let $x=x^2$, then $x^2+5x+6=(x+3)(x+2)$.

<u>Correct Solution</u>. $x^4+5x^2+6=(x^2)^2+5(x^2)+6$. Let $u=x^2$, then $u^2+5u+6=(u+3)(u+2)=(x^2+3)(x^2+2)$.

8. $49x^2-144=$

<u>Wrong Solution.</u> $(7x-12)^2$.

<u>Correct Solution</u>. $(7x-12)(7x+12)$.

9. $x^2+4x+4=$

<u>Wrong Solution.</u> $(x+2)(x-2)$.

<u>Correct Solution</u>. $(x+2)(x+2)=(x+2)^2$.

10. $x^3+8=$

<u>Wrong Solution.</u> $(x+2)(x^2+2x+4)$.

<u>Correct Solution</u>. $(x+2)(x^2-2x+4)$.

11. $x^3+8=$

<u>Wrong Solution.</u> $(x+2)(x^2-4x+4)$.

<u>Correct Solution</u>. $(x+2)(x^2-2x+4)$.

12. $x^3+8=$

 <u>Wrong Solution</u>. $x^3+(2)^3=(x+2)^3$.

 <u>Correct Solution</u>. $(x+2)(x^2-2x+4)$.

13. $a^2+10ab+25b^2-c^2=$

 <u>Wrong Solution</u>. $a(a+10b)+(5b-c)(5b+c)$.

 <u>Correct Solution</u>. $(a^2+10ab+25b^2)-c^2=(a+5b)^2-c^2=(a+5b-c)(a+5b+c)$.

14. $x^6+7x^3-8=$

 <u>Wrong Solution</u>. $(x^3+8)(x^3-1)$.

 <u>Correct Solution</u>. $(x^3+8)(x^3-1)=(x+2)(x^2-2x+4)(x-1)(x^2+x+1)$.

15. $p^2-p+q-q^2=$

 <u>Wrong Solution</u>. $p^2-q^2-p+q=(p-q)(p+q)-(p+q)=(p+q)(p-q-1)$.

 <u>Correct Solution</u>. $p^2-q^2-p+q=(p-q)(p+q)-(p-q)=(p-q)(p+q-1)$.

16. $x^{4n}-3x^{2n}-4=$

 <u>Wrong Solution</u>. Let $y=x^{2n}$, then $x^{4n}-3x^{2n}-4=y^{2n}-3y-4$.

 <u>Correct Solution</u>. Let $y=x^{2n}$, then $x^{4n}-3x^{2n}-4=y^2-3y-4=(y-4)(y+1)=$ $(x^{2n}-4)(x^{2n}+1)=(x^n-2)(x^n+2)(x^{2n}+1)$.

17. $x^2+3x-4=$

 <u>Wrong Solution</u>. Let $x^2+3x-4=0$, then $x^2+3x=4$.

 <u>Correct Solution</u>. $x^2+3x-4=(x+4)(x-1)$.

P.5 Rational Expressions

Objectives

To introduce

- How to Simplify a Rational Expression
- Operations on Rational Expressions
- How to Determine the Least Common Denominator (LCD) of Rational Expressions
- How to Simplify Complex Fractions

I. **True or False Statements (Give a Reason):**

1. $\dfrac{P}{Q} = \dfrac{R}{S}$ means $P = R$ and $Q = S$

   Wrong Answer. True.

   Correct Answer. False, since $\dfrac{1}{2} = \dfrac{2}{4}$ but $1 \neq 2$.

2. $\dfrac{P}{Q} = \dfrac{PR}{QR}$ for all real numbers R.

   Wrong Answer. True.

   Correct Answer. False, since if $R = 0$, then $\dfrac{PR}{QR}$ is indeterminate expression

3. $\dfrac{-P}{Q} = \dfrac{-P}{-Q}$.

   Wrong Answer. True.

   Correct Answer. False, since $\dfrac{-P}{-Q} = \dfrac{P}{Q}$.

4. The domain of $\dfrac{4}{x^2+1}$ is all real numbers except -1 and 1.

   Wrong Answer. True.

   Correct Answer. False, since $x^2 + 1 \neq 0$ is true for all real numbers and so the domain is all real numbers.

5. $\frac{x-3}{x+3} = -1.$

Wrong Answer. True.

Correct Answer. False, since if $x = 3$, then $\frac{x-3}{x+3} = 0$ and not -1.

6. $\frac{x+3}{3} = x + 1.$

Wrong Answer. True.

Correct Answer. False, since $\frac{x+3}{3} = \frac{x}{3} + \frac{3}{3} = \frac{x}{3} + 1.$

7. $\frac{x}{x+3} = 1 + \frac{x}{3}.$

Wrong Answer. True.

Correct Answer. False, since if $x = 3$, then $\frac{x}{x+3} = \frac{1}{2}$ but $1 + \frac{x}{3} = 2$.

8. $\frac{P}{Q} \pm \frac{R}{S} = \frac{P \pm R}{Q \pm S}.$

Wrong Answer. True.

Correct Answer. False, since $\frac{P}{Q} \pm \frac{R}{S} = \frac{PS \pm RQ}{QS}.$

9. $\frac{P}{Q} \pm \frac{R}{S} = \frac{P \pm R}{QS}.$

Wrong Answer. True.

Correct Answer. False, since $\frac{P}{Q} \pm \frac{R}{S} = \frac{PS \pm RQ}{QS}.$

10. $\frac{P}{Q} \div \frac{R}{S} = \frac{Q}{P} \cdot \frac{R}{S}.$

Wrong Answer. True.

Correct Answer. False, since $\frac{P}{Q} \div \frac{R}{S} = \frac{P}{Q} \cdot \frac{S}{R}.$

11. $\frac{x-7}{7-x} = 1.$

Wrong Answer. True.

Correct Answer. False, since $\frac{x-7}{7-x} = \frac{x-7}{-(x-7)} = -1$.

12. $\frac{xy+3x-2}{x} = y+3x-2$.

Wrong Answer. True.

Correct Answer. False, since $\frac{xy+3x-2}{x} = y+3-\frac{2}{x}$.

13. $\frac{xy+3x-2}{x} = y+3-2 = y+1$.

Wrong Answer. True.

Correct Answer. False, since $\frac{xy+3x-2}{x} = y+3-\frac{2}{x}$.

14. $2x^{-1}y = \frac{y}{2x}$.

Wrong Answer. True.

Correct Answer. False, since $2x^{-1}y = \frac{2y}{x}$.

15. $xy^{-1} = \frac{1}{xy}$.

Wrong Answer. True.

Correct Answer. False, since $xy^{-1} = \frac{x}{y}$.

16. $a^{-2} = \frac{2}{a}$.

Wrong Answer. True.

Correct Answer. False, since $a^{-2} = \frac{1}{a^2}$.

17. $\frac{a-c}{a} = -c$.

Wrong Answer. True.

Correct Answer. False, since $\frac{a-c}{a} = \frac{a}{a} - \frac{c}{a} = 1 - \frac{c}{a}$.

18. $ab^{-1} - a^{-1}b = ab(b^{-1} - a^{-1})$.

Wrong Answer. True.

Correct Answer. False, since $ab^{-1} - a^{-1}b = a^{-1}b^{-1}(a^2 - b^2) = a^{-1}b^{-1}(a - b)(a + b)$.

19. $\dfrac{a^3 - b^3}{a^2 - b^2} = a - b$.

Wrong Answer. True.

Correct Answer. False, since $\dfrac{a^3 - b^3}{a^2 - b^2} = \dfrac{(a - b)(a^2 + ab + b^2)}{(a - b)(a + b)} = \dfrac{a^2 + ab + b^2}{a + b}$.

20. $\dfrac{ab}{c} - \dfrac{b}{d} = \dfrac{a}{c} - \dfrac{1}{d}$.

Wrong Answer. True.

Correct Answer. False, since $\dfrac{ab}{c} - \dfrac{b}{d} = \dfrac{abd - bc}{dc}$.

21. $(x + y)^{-1} = x^{-1} + y^{-1}$.

Wrong Answer. True.

Correct Answer. False, since if $x = 1$ and $y = 2$, then $(x + y)^{-1} = \dfrac{1}{3}$ but $x^{-1} + y^{-1} = 1 + \dfrac{1}{2} = \dfrac{3}{2}$.

22. $\dfrac{y + x}{y^2 - x^2} = \dfrac{y}{y^2} - \dfrac{x}{x^2} = \dfrac{1}{y} - \dfrac{1}{x}$.

Wrong Answer. True.

Correct Answer. False, since $\dfrac{y + x}{y^2 - x^2} = \dfrac{(y + x)}{(y - x)(y + x)} = \dfrac{1}{y - x}$.

23. $\dfrac{1}{x + 2} = \dfrac{1}{2x}$.

Wrong Answer. True.

Correct Answer. False, since if $x = 1$, then $\dfrac{1}{x + 2} = \dfrac{1}{3}$ but $\dfrac{1}{2x} = \dfrac{1}{2}$.

24. $\dfrac{a}{b} \cdot \dfrac{c}{b} = \dfrac{ac}{b}$.

Wrong Answer. True.

Correct Answer. False, since $\frac{a}{b} \cdot \frac{c}{b} = \frac{ac}{b^2}$.

25. $\dfrac{a+b+c+d}{a+c} = b+d$.

Wrong Answer. True.

Correct Answer. False, since $\dfrac{(a+b)+(c+d)}{(a+b)} = 1 + \dfrac{c+d}{a+b}$.

26. $x^{-1} - y^{-1} = \dfrac{1}{x-y}$.

Wrong Answer. True.

Correct Answer. False, since $x^{-1} - y^{-1} = \dfrac{1}{x} - \dfrac{1}{y} = \dfrac{y-x}{xy}$.

27. $(x+y)(x^2y^2) = (x+y)^3$.

Wrong Answer. True.

Correct Answer. False, since $(x+y)(x^2y^2) = x^3y^2 + x^2y^3$.

28. $\dfrac{b^{-1} - a^{-1}}{b^{-2} - a^{-2}} = \dfrac{b^2 - a^2}{b-a}$.

Wrong Answer. True.

Correct Answer. False, since $\dfrac{b^{-1} - a^{-1}}{b^{-2} - a^{-2}} = \dfrac{\frac{1}{b} - \frac{1}{a}}{\frac{1}{b^2} - \frac{1}{a^2}} = \dfrac{\frac{a-b}{ab}}{\frac{a^2-b^2}{b^2a^2}} = \dfrac{a-b}{ab} \cdot$

$\dfrac{b^2a^2}{(a-b)(a+b)} = \dfrac{ba}{a+b}$.

29. $\dfrac{x^{3n} + y^{6m}}{x^n + y^{2m}} = x^{2n} + y^{4m}$.

Wrong Answer. True.

Correct Answer. False, since $\dfrac{a^3 + b^6}{a + b^2} \neq a^2 + b^4$.

30. $\frac{a}{b} \div \left[\frac{c}{d} - \frac{e}{f}\right] = \frac{a}{b} \cdot \left[\frac{d}{c} - \frac{f}{e}\right]$.

Wrong Answer. True.

Correct Answer. False, since $\frac{a}{b} \div \left(\dfrac{cf - ed}{df}\right) = \frac{a}{b} \cdot \dfrac{df}{cf - ed}$.

31. $\dfrac{x^2+x-6}{x^2-5x+6} = \dfrac{x}{5x} = \dfrac{1}{5}$.

Wrong Answer. True.

Correct Answer. False, since $\dfrac{x^2+x-6}{x^2-5x+6} = \dfrac{(x+3)(x-2)}{(x-3)(x-2)} = \dfrac{x+3}{x-3}$.

32. $xy(x+y)^{-1} = (x^2y+xy^2)^{-1}$.

Wrong Answer. True.

Correct Answer. False, since $xy(x+y)^{-1} = \dfrac{xy}{x+y}$.

33. $\dfrac{\frac{1}{5}}{2} = \dfrac{2}{5}$.

Wrong Answer. True.

Correct Answer. False, since $\dfrac{\frac{1}{5}}{2} = \dfrac{1}{5}\cdot\dfrac{1}{2} = \dfrac{1}{10}$.

34. The LCD of $\dfrac{1}{18}$ and $\dfrac{1}{9}$ is 9.

Wrong Answer. True.

Correct Answer. False, since the LCD is 18.

35. The LCD of $\dfrac{5x}{(x+5)}$ and $\dfrac{7}{x(x+5)}$ is $(x+5)$.

Wrong Answer. True.

Correct Answer. False, since the LCD is $x(x+5)$.

36. The LCD of $\dfrac{5}{(x-7)^2}$ and $\dfrac{3}{(x-7)}$ is $(x-7)$.

Wrong Answer. True.

Correct Answer. False, since the LCD is $(x-7)^2$.

37. The LCD of $\dfrac{1}{(x+1)(x+2)}$ and $\dfrac{1}{(x+2)(x+6)}$ is $(x+1)(x+2)^2(x+6)$.

Wrong Answer. True.

Correct Answer. False, since the LCD is $(x+1)(x+2)(x+6)$.

38. $\dfrac{c^{-1}}{a^{-1}+b^{-1}} = \dfrac{a+b}{c}$.

Wrong Answer. True.

Correct Answer. False, since $\dfrac{c^{-1}}{a^{-1}+b^{-1}} = \dfrac{\frac{1}{c}}{\frac{1}{a}+\frac{1}{b}} = \dfrac{\frac{1}{c}}{\frac{b+a}{ab}} = \dfrac{1}{c}\cdot\dfrac{ab}{b+a} = \dfrac{ab}{c(b+a)}$.

39. $\dfrac{a}{b}\cdot\dfrac{c}{d} = \dfrac{ac}{bdb}$.

Wrong Answer. True.

Correct Answer. False, since $\dfrac{a}{b}\cdot\dfrac{c}{d} = \dfrac{ac}{bd}$.

40. $\dfrac{1}{2+\frac{1}{x}} = \dfrac{x}{2+1} = \dfrac{x}{3}$.

Wrong Answer. True.

Correct Answer. False, since $\dfrac{1}{2+\frac{1}{x}} = \dfrac{1}{\frac{2x+1}{x}} = \dfrac{x}{2x+1}$.

41. $\dfrac{a}{b}\left(\dfrac{1}{c}-\dfrac{1}{d}\right) = \dfrac{a}{bc}-\dfrac{1}{d}$.

Wrong Answer. True.

Correct Answer. False, since $\dfrac{a}{b}\left(\dfrac{1}{c}-\dfrac{1}{d}\right) = \dfrac{a}{bc}-\dfrac{a}{bd}$.

II. **Solutions with Wrong Steps**:

Simplify each of the following rational expressions:

1. $\dfrac{3x-15}{x^2-25}\cdot\dfrac{x^2+8x+15}{6x+9}$.

Wrong Solution. $\dfrac{3x-15}{x^2-25}\cdot\dfrac{x^2+8x+15}{6x+9} = \dfrac{3x^3+9x^2-75x-225}{6x^3+9x^2-150x-225}$.

Correct Solution. $\dfrac{3x-15}{x^2-25}\cdot\dfrac{x^2+8x+15}{6x+9} = \dfrac{3(x-5)}{(x-5)(x+5)}\cdot\dfrac{(x+5)(x+3)}{3(2x+3)} = \dfrac{x+3}{2x+3}$.

2. $\frac{x}{x^2-4} - \frac{2x-1}{x^2-3x-10}$.

Wrong Solution. $\frac{x}{x^2-4} - \frac{2x-1}{x^2-3x-10} = \frac{x(x^2-3x-10)-(2x-1)(x^2-4)}{(x^2-4)(x^2-3x-10)}$.

Correct Solution. $\frac{x}{x^2-4} - \frac{2x-1}{x^2-3x-10} = \frac{x}{(x-2)(x+2)} - \frac{2x-1}{(x-5)(x+2)}$
$= \frac{x(x-5)-(2x-1)(x-2)}{(x-2)(x+2)(x-5)} = \frac{x^2-5x-2x^2+5x-2}{(x-2)(x+2)(x-5)} = \frac{-x^2-2}{(x-2)(x+2)(x-5)}$.

3. $\frac{2}{y} - \frac{3}{y+1} \cdot \frac{y^2-1}{y+4}$.

Wrong Solution. $\frac{2}{y} - \frac{3}{y+1} \cdot \frac{y^2-1}{y+4} = \frac{2(y+1)-3y}{y(y+1)} \cdot \frac{y^2-1}{y+4}$.

Correct Solution. $\frac{2}{y} - \frac{3}{y+1} \cdot \frac{y^2-1}{y+4} = \frac{2}{y} - \frac{3}{y+1} \cdot \frac{(y-1)(y+1)}{y+4} = \frac{2}{y} - \frac{3(y-1)}{y+4} =$
$\frac{2(y+4)-3y(y-1)}{y(y+4)} = \frac{2y+8-3y^2+3y}{y(y+4)} = \frac{-3y^2+5y+8}{y(y+4)}$.

4. $\frac{1}{x+1} - \frac{2}{x}$.

Wrong Solution. $\frac{1}{x+1} - \frac{2}{x} = \frac{x-2x+1}{x(x+1)} = \frac{1-x}{x(x+1)}$.

Correct Solution. $\frac{1}{x+1} - \frac{2}{x} = \frac{x-2(x+1)}{x(x+1)} = \frac{x-2x-2}{x(x+1)} = \frac{-x-2}{x(x+1)}$.

P.6 Complex Numbers

Objectives

To introduce

- Complex Numbers
- Operations on Complex numbers
- Complex Conjugates

I. True or False Statements (Give a Reason):

1. $\sqrt{-4}$ is a real number.

   Wrong Answer. True.

   Correct Answer. False, since $\sqrt{-4} = 2i$ which is not a real number.

2. The real part of $2i + 1$ is 2.

   Wrong Answer. True.

   Correct Answer. False, since $2i + 1 = 1 + 2i$ and so the real part is 1.

3. 3 is not a complex number.

   Wrong Answer. True.

   Correct Answer. False, since 3 can be written as $3 + 0i$.

4. 0 is an imaginary number.

   Wrong Answer. True, since $0 = 0i$.

   Correct Answer. False, since 0 is a real number.

5. i^2 is an imaginary number.

   Wrong Answer. True.

Correct Answer. False, since $i^2 = -1$ which is a real number.

6. $\sqrt{-2}\sqrt{-8} = 4$.

Wrong Answer. True, since $\sqrt{-2}\sqrt{-8} = \sqrt{(-2)(-8)} = \sqrt{16} = 4$.

Correct Answer. False, since $\sqrt{-2}\sqrt{-8} = i\sqrt{2}i\sqrt{8} = i^2\sqrt{16} = -4$.

7. $(a+bi)(c+di) = (ac-bd)$.

Wrong Answer. True.

Correct Answer. False, since $(a+bi)(c+di) = (ac-bd) + (ad+bc)i$.

8. The conjugate of $2i - 1$ is $2i + 1$.

Wrong Answer. True.

Correct Answer. False, since $2i - 1$ is $-1 + 2i$ where its conjugate is $-1 - 2i$.

9. $i^{-3} = i^3 = -i$.

Wrong Answer. True.

Correct Answer. False, since $i^{-3} = \dfrac{1}{i^3} = \dfrac{1}{-i} \cdot \dfrac{i}{i} = \dfrac{i}{-i^2} = i$.

10. $(a-bi)^2 = a^2 + b^2$.

Wrong Answer. True.

Correct Answer. False, since $(a-bi)^2 = a^2 - 2abi + b^2i^2 = a^2 - b^2 - 2abi$.

11. $(a-bi)(a+bi) = a^2 - b^2$.

Wrong Answer. True.

Correct Answer. False, since $(a-bi)(a+bi) = a^2 - b^2i^2 = a^2 + b^2$.

12. $i + 2$ is in standard form.

Wrong Answer. True.

<u>Correct Answer</u>. <u>False</u>, since $i+2=2+i$ and $2+i$ is the standard form.

13. $\sqrt{-2}\sqrt{-8}=4i$.

<u>Wrong Answer</u>. <u>True</u>.

<u>Correct Answer</u>. <u>False</u>, since $\sqrt{-2}\sqrt{-8}=i\sqrt{2}\,i\sqrt{8}=i^2\sqrt{16}=-4$.

14. $7i=7\sqrt{-1}=-\sqrt{7}$.

<u>Wrong Answer</u>. <u>True</u>.

<u>Correct Answer</u>. <u>False</u>, since $7\sqrt{-1}\neq-\sqrt{7}$.

15. $2-i=2+1=3$.

<u>Wrong Answer</u>. <u>True</u>, since $2-i=2-\sqrt{-1}=2+\sqrt{1}=2+1=3$.

<u>Correct Answer</u>. <u>False</u>, since $2-i=2-\sqrt{-1}$.

16. $\dfrac{6+5i}{3-i}=\dfrac{6}{3}-\dfrac{5}{1}i=2-5i$.

<u>Wrong Answer</u>. <u>True</u>.

<u>Correct Answer</u>. <u>False</u>, since $\dfrac{6+5i}{3-i}\cdot\dfrac{3+i}{3+i}=\dfrac{13+21i}{10}=\dfrac{13}{10}+\dfrac{21}{10}i$.

17. $\dfrac{6+5i}{3-i}=\dfrac{6}{3}-\dfrac{5i}{i}=2-5=-3$.

<u>Wrong Answer</u>. <u>True</u>.

<u>Correct Answer</u>. <u>False</u>, see (16).

18. $1+2i=3i$.

<u>Wrong Answer</u>. <u>True</u>, since $1+2i=(1+2)\,i=3i$.

<u>Correct Answer</u>. <u>False</u>, since $1+2i=1+2\sqrt{-1}$.

19. $-a+ai=0$.

Wrong Answer. True, since $-a + ai = (-a + a)\,i = 0i = 0$.

Correct Answer. False, since $-a + ai = a(-1 + i)$.

20. The conjugate of $-1 + i$ is $1 - i$.

Wrong Answer. True.

Correct Answer. False, since the conjugate of $-1 + i$ is $-1 - i$.

21. $i^3 = -1$.

Wrong Answer. True.

Correct Answer. False, since $i^3 = i^2 \cdot i = -i$.

22. $6i = -6$.

Wrong Answer. True.

Correct Answer. False, since $6i$ is an imaginary number while -6 is a real number.

Chapter 1

Equations And Inequalities

Objectives

To introduce

- Linear and Absolute Value Equations
- Formulas and Applications
- Quadratic Equations
- Other Types of Equations
- Inequalities

1.1 Linear and Absolute Value Equations

Objectives

To introduce

- Linear Equations
- Contradictions, Conditional Equations, and Identities
- Absolute Value Equations

Solutions with Wrong Steps:

1. Solve: $\frac{2}{3}x - 5 = \frac{1}{2}x - 3$.

   Wrong Solution. Multiply both sides by $6 \Rightarrow 4x - 5 = 3x - 3 \Rightarrow x = 2$.

   Correct Solution. Multiply both sides by $6 \Rightarrow 4x - 30 = 3x - 18 \Rightarrow x = 12$.

2. Solve: $(x+2)(3x-1) = (3x+1)(x+2)$.

   Wrong Solution. Divide both sides by $x + 2 \Rightarrow 3x - 1 = 3x + 1 \Rightarrow 2 = 0 \Rightarrow$ the equation is a contradiction.

   Correct Solution. $\Rightarrow 3x^2 + 5x - 2 = 3x^2 + 7x + 2 \Rightarrow 5x - 2 = 7x + 2 \Rightarrow -4 = 2x \Rightarrow x = -2$ and the equation is conditional.

3. Solve: $3[x - 2(x-5)] = -3x + 30$.

   Wrong Solution. $\Rightarrow 3[x-2x-10] = -3x+30 \Rightarrow 3[-x-10] = -3x+30 \Rightarrow -3x-10 = -3x + 30 \Rightarrow -10 = 30 \Rightarrow$ the equation is a contradiction.

   Correct Solution. $\Rightarrow 3[x - 2x + 10] = -3x + 30 \Rightarrow 3[-x + 10] = -3x + 30 \Rightarrow -3x + 30 = -3x + 30 \Rightarrow$ The original equation is true for any number x. The equation is an identity.

4. Solve: $|x-1|=-4$

   Wrong Solution. $x-1=-4$ or $x-1=4 \Rightarrow x=-3$ or $x=5$.

   Correct Solution. We know that $|x-1| \geq 0$, thus the given equation is a contradiction.

5. Solve: $|-x+4|=8$.

   Wrong Solution. $\Rightarrow -(-x+4)=8 \Rightarrow x-4=8 \Rightarrow x=12$.

   Correct Solution. $\Rightarrow -x+4=8$ or $-x+4=-8 \Rightarrow x=-4$ or $x=12$.

6. Solve: $|7-x|=|x-7|$.

   Wrong Solution. $\Rightarrow 7-x=-x-7 \Rightarrow 14=0$ and the equation is a contradiction.

   Correct Solution. $LHS=|7-x|=|-(-7+x)|=|-7+x|=|x-7|=RHS \Rightarrow$ the equation is an identity (Notice that $|a-b|=|b-a|$).

7. Solve: $|x+4|=x+4$.

   Wrong Solution. $x+4=-(x+4)$ or $x+4=x+4 \Rightarrow x+4=-x-4$ or x is any real number $\Rightarrow x$ is any real number.

   Correct Solution. Since $|x+4| \geq 0 \Rightarrow x+4 \geq 0 \Rightarrow x \geq -4 \Rightarrow$ The statement is true for any $x \geq -4$.

1.2 Formulas and Applications

Objectives

- To Solve a Formula for a Specified Variable
- To Introduce Strategies for Solving Word Problems

Solutions with Wrong Steps:

1. If $P = \dfrac{S}{S+F}$, solve for S.

   Wrong Solution. $\Rightarrow P = 1 + \dfrac{S}{F} \Rightarrow P - 1 = \dfrac{S}{F} \Rightarrow S = F(P-1)$.

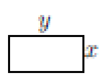
   Correct Solution. $\Rightarrow P(S+F) = S \Rightarrow PS + PF = S \Rightarrow PF = S - PS = S(1-P) \Rightarrow S = \dfrac{PF}{1-P}$.

2. If $x - y = xy + 1$, solve for x.

   Wrong Solution. $x = y + xy + 1$.

   Correct Solution. $x - xy = y + 1 \Rightarrow x(1-y) = y + 1 \Rightarrow x = \dfrac{y+1}{1-y}$.

3. The width of a rectangle is 1 meter more than half the length of the rectangle. If the perimeter of the rectangle is 110 meters, find the length of the rectangle.



   Wrong Solution. $x =$ width , $y =$ length $\Rightarrow x + 1 = \dfrac{1}{2}y,\ \ x + y = 55 \Rightarrow (x+y) - (x+1) = 55 - \dfrac{1}{2}y \Rightarrow y - 1 = 55 - \dfrac{1}{2}y \Rightarrow \dfrac{3}{2}y = 56 \Rightarrow y = \dfrac{(56)(2)}{3} = \dfrac{112}{3}$.

   Correct Solution. $x = \dfrac{1}{2}y + 1,\ \ x + y = 55 \Rightarrow (x+y) - x = 55 - \left(\dfrac{1}{2}y + 1\right) \Rightarrow y = 54 - \dfrac{1}{2}y \Rightarrow \dfrac{3}{2}y = 54 \Rightarrow y = \dfrac{(2)(54)}{3} = (2)(18) = 36$.

1.3 Quadratic Equations

Objectives:

- To Solve Quadratic Equations by Factoring, Square Root Procedure, Completing the Square, and by Using the Qudratic Formula
- To Introduce the Discriminant of a Quadratic Equation and Use it to Determine the Number of Real Solutions

Solutions with Wrong Steps:

1. Solve the quadratic equation by factoring:

$$3x^2 + 13x = 10.$$

<u>Wrong Solution</u>. $\Rightarrow x(3x+13) = 10 \Rightarrow x = 10$ or $3x + 13 = 10 \Rightarrow x = 10$ or $3x = -3 \Rightarrow x = 10$ or $x = -1$.

<u>Correct Solution</u>. $\Rightarrow 3x^2 + 13x - 10 = 0 \Rightarrow (3x-2)(x+5) = 0 \Rightarrow 3x - 2 = 0$ or $x + 5 = 0 \Rightarrow x = \frac{2}{3}$ or $x = -5$.

2. Use the square root procedure to solve the quadratic equation $(2-3x)^2 = 16$.

<u>Wrong Solution</u>. $\Rightarrow -(3x-2)^2 = 16 \Rightarrow (3x-2)^2 = -16 \Rightarrow 3x - 2 = \pm\sqrt{-16} \Rightarrow 3x = 2 \pm 4i \Rightarrow x = \dfrac{2 \pm 4i}{3}$.

<u>Correct Solution</u>. $\Rightarrow 2 - 3x = \pm\sqrt{16} = \pm 4 \Rightarrow 2 - 3x = 4$ or $2 - 3x = -4 \Rightarrow 3x = -2$ or $3x = 6 \Rightarrow x = -\frac{2}{3}$ or $x = 2$.

3. Solve $3x^2 + 6x - 2 = 0$ by completing the square.

<u>Wrong Solution</u>. $\Rightarrow 3x^2 + 6x = 2 \Rightarrow x^2 + 2x = 2 \Rightarrow (x+1)^2 - 1 = 2 \Rightarrow (x+1)^2 = 3$ $\Rightarrow x + 1 = -\sqrt{3}$ or $x + 1 = \sqrt{3} \Rightarrow x = -1 - \sqrt{3}$ or $x = -1 + \sqrt{3}$.

Correct Solution. $\Rightarrow 3x^2+6x=2 \Rightarrow x^2+2x=\frac{2}{3} \Rightarrow (x+1)^2-1=\frac{2}{3} \Rightarrow (x+1)^2=\frac{5}{3}$
$\Rightarrow x+1=-\sqrt{\frac{5}{3}}=-\frac{\sqrt{15}}{3}$ or $x+1=\sqrt{\frac{5}{3}}=\frac{\sqrt{15}}{3} \Rightarrow x=-1-\frac{\sqrt{15}}{3}$ or $x=-1+\frac{\sqrt{15}}{3}$.

4. Solve $-2x^2+3x+1=0$ by using the quadratic formula.

Wrong Solution. $\Rightarrow 2x^2+3x+1=0 \Rightarrow a=2, b=3, c=1$ and $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \Rightarrow$
$x=\frac{-3\pm\sqrt{9-8}}{4}=\frac{-3\pm 1}{4} \Rightarrow x=-\frac{1}{2}$ or $x=-1$.

Correct Solution. $a=-2,\ b=3,\ c=1,\ x=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \Rightarrow x=\frac{-3\pm\sqrt{9+8}}{-4} \Rightarrow$
$x=\frac{-3\pm\sqrt{17}}{-4} \Rightarrow x=\frac{3-\sqrt{17}}{4}$ or $x=\frac{3+\sqrt{17}}{4}$.

5. Find the discriminant of the equation $5x^2-3x-2=0$.

Wrong Solution. $a=5,\ b=-3,\ c=-2$ and the discriminant $=\sqrt{b^2-4ac}=$
$\sqrt{9+40}=\sqrt{49}=\pm 7$.

Correct Solution. $a=5,\ b=-3,\ c=-2$ and the discriminant $=b^2-4ac=9+40=49$.

6. Find the discriminant of the equation $5x^2-3x=2$.

Wrong Solution. $a=5,\ b=-3,\ c=2$ and the discriminant $=b^2-4ac=9-40=$
-31.

Correct Solution. $\Rightarrow 5x^2-3x-2=0 \Rightarrow b^2-4ac=49$ as in (5).

1.4 Other Types of Equations

Objectives

To introduce

- Polynomial Equations
- Rational Equations
- Radical Equations
- Equations that are Quadratic in Form

Solutions with Wrong Steps:

Solve each of the following Equation:

1. $x^3 = x$.

   Wrong Solution. By dividing by x, we will get $x^2 = 1$ and so the solution set is $\{-1, 1\}$.

   Correct Solution. $x^3 = x$ can be solved in this way:

$$x^3 - x = 0$$
$$x(x^2 - 1) = 0$$
$$x(x-1)(x+1) = 0$$

   So, the solution set is $\{-1, 0, 1\}$.

2. $1 + \frac{x}{x-5} = \frac{5}{x-5}$.

   Wrong Solution. If we multiply the equation by $x - 5$, we get

$$x - 5 + x = 5$$
$$2x = 10$$
$$x = 5$$

So, the solution set is $\{5\}$.

Correct Solution. Same as above except that 5 is not a solution since substituting 5 for x in the original equation results in denominators of 0 and so the original equation has no solution.

3. $\sqrt{x} = -1$.

Wrong Solution. Square both sides to get $x = 1$ and so the solution set is $\{1\}$.

Correct Solution. Same as above except that by substituting 1 in the original equation to check the answer, we will get $1 = -1$ which is wrong and so the original equation has no solution.

4. $x = 2 + \sqrt{2 - x}$.

Wrong Solution. Squaring both sides by squaring each term, we get $x^2 = 4 + 2 - x \Rightarrow x^2 + x - 6 = 0 \Rightarrow (x+3)(x-2) = 0$. So, we get $\{-3, 2\}$ is the solution set.

Correct Solution.

$$\begin{aligned} x &= 2 + \sqrt{2-x} \\ \Rightarrow x - 2 &= \sqrt{2-x} \\ \Rightarrow (x-2)^2 &= 2 - x \Rightarrow x^2 - 4x + 4 = 2 - x \Rightarrow x^2 - 3x + 2 = 0 \\ &\Rightarrow (x-2)(x-1) = 0 \Rightarrow x = 2 \text{ or } x = 1 \end{aligned}$$

2 checks as a solution, but 1 does not. Therefore, 2 is the only solution.

5. $\sqrt{x+1} - \sqrt{2x-5} = 1$.

Wrong Solution. By squaring both sides, we get $x + 1 - (2x - 5) = 1 \Rightarrow -x + 5 = 0 \Rightarrow x = 5$.

Correct Solution.

$$\begin{aligned}
\sqrt{x+1} &= 1+\sqrt{2x-5} \quad \text{Square both sides}\\
\Rightarrow (\sqrt{x+1})^2 &= (1+\sqrt{2x-5})^2\\
\Rightarrow x+1 &= 1+2\sqrt{2x-5}+(2x-5)\\
\Rightarrow x+1 &= 2x-4+2\sqrt{2x-5}\\
\Rightarrow -x+5 &= 2\sqrt{2x-5} \quad \text{Square both sides again}\\
\Rightarrow x^2-10x+25 &= 4(2x-5)\\
\Rightarrow x^2-18x+45 &= 0\\
\Rightarrow (x-15)(x-3) &= 0\\
\Rightarrow x &= 15 \text{ or } x=3.
\end{aligned}$$

3 checks as a solution, but 15 does not. Therefore, 3 is the only solution.

6. $\sqrt{x+1}=\sqrt{2x-5}+1$.

Wrong Solution.

$$\begin{aligned}
\sqrt{x+1} &= \sqrt{2x-5}+1 \qquad \text{Square both sides}\\
\Rightarrow x+1 &= 2x-5+2\sqrt{2x-5}+1\\
\Rightarrow x+1 &= 2x-4+2\sqrt{2x-5}
\end{aligned}$$

Some students will square both sides without rearranging the equation.

Correct Solution. Follow the same correct steps in (5).

7. $x=2\sqrt{x}$.

Wrong Solution.

$$\begin{aligned}
x &= 2\sqrt{x} \qquad \text{Square both sides}\\
\Rightarrow x^2 &= 2(\sqrt{x})^2\\
\Rightarrow x^2 &= 2(x) \Rightarrow x(x-2)=0 \Rightarrow x=0 \text{ or } x=2 \Rightarrow .
\end{aligned}$$

The solution set is $\{0, 2\}$.

Correct Solution.

$$\begin{aligned} x &= 2\sqrt{x} \\ x^2 &= (2\sqrt{x})^2 = 4x \\ x^2 - 4x &= 0 \\ x(x-4) &= 0 \\ x &= 0 \text{ or } x = 4. \end{aligned}$$

Check that both 0 and 4 are solutions and so the solution set is $\{0, 4\}$.

8. $x^4 - 3x^2 + 2 = 0$.

Wrong Solution.

$$\begin{aligned} x^4 - 3x^2 + 2 &= 0 \qquad \text{Let } x^2 = x \\ x^2 - 3x + 2 &= 0 \\ (x-2)(x-1) &= 0 \\ x &= 2 \text{ or } x = 1. \end{aligned}$$

The solution set is $\{1, 2\}$.

Correct Solution.

$$\begin{aligned} x^4 - 3x^2 + 2 &= 0 \qquad \text{Let } u = x^2 \\ u^2 - 3u + 2 &= 0 \\ (u-2)(u-1) &= 0 \\ u &= 2 \text{ or } u = 1 \\ x^2 &= 2 \text{ or } x^2 = 1 \\ x &= \pm\sqrt{2} \text{ or } x = \pm 1. \end{aligned}$$

The solution set is $\{-\sqrt{2}, -1, 1, \sqrt{2}\}$.

9. $x^4 - 3x^2 + 2 = 0$.

Wrong Solution.

$$\begin{aligned} x^4 - 3x^2 + 2 &= 0 \qquad \text{Let } u = x^2. \\ u^2 - 3u + 2 &= 0 \\ (u-2)(u-1) &= 0 \\ u = 2 \text{ or } u = 1. \end{aligned}$$

The solution set is $\{1, 2\}$.

Correct Solution. See the correct solution given in (8).

10. $x^{1/3} = -1$.

Wrong Solution. No solution since $x^{1/3}$ is always positive.

Correct Solution. We cube both sides of the equation to get $x = -1$ and so the solution set is $\{-1\}$. This is true since $x^{1/3}$ can be a negative number.

1.5 Inequalities

Objectives

To introduce

- Properties of Inequalities
- Compound Inequalities
- Absolute Value Inequalities
- The Critical Value Method
- Rational Inequalities

I. **True or False Statements (Give a Reason):**

1. If $x < 3$, then $x^2 < 3x$.

   Wrong Solution. True, since x is a positive number.

   Correct Solution. False, since x may be a positive or a negative number.

2. If $x + 3 > 0$, then $x > 3$.

   Wrong Solution. True.

   Correct Solution. False, since $x + 3 > 0$ implies that $x > -3$.

3. If $x < -3$ or $x > 5$, then $-3 > x > 5$.

   Wrong Solution. True.

   Correct Solution. False, since $-3 > 5$ is a wrong statement.

4. $x < 3$ and $x > 1$ is the same as $x < 3$ or $x > 1$.

   Wrong Solution. True.

Correct Solution. False, since "and" means the intersection of the two sets, which is $(1, 3)$ while or means the union of the two sets, which is $(-\infty, \infty)$.

5. The solution set of $2 > x > 5$ is $(2, 5)$.

   Wrong Solution. True.

   Correct Solution. False, since there is no number less than 2 and greater than 5.

6. If $|x| < k$, then $-k < x < k$ for any real number k.

   Wrong Solution. True.

   Correct Solution. False, since this is true only if k is a positive real number.

7. If $|x| > k$, then $x < -k$ or $x > k$ for any real number k.

   Wrong Solution. True.

   Correct Solution. False, since this is true only if k is a positive real number.

8. If $|x| > k$, then $-k > x > k$ if k is a positive real number.

   Wrong Solution. True.

   Correct Solution. False, if $k > 0$, then $-k > k$ is a wrong statement.

9. If $\frac{1}{x} < 2$, then $x < \frac{1}{2}$.

   Wrong Solution. True.

   Correct Solution. False, since if $x = 1$, then it will satsify $\frac{1}{x} < 2$, but not $x < \frac{1}{2}$.

10. If $\frac{1}{x} < 2$, then $x > \frac{1}{2}$.

    Wrong Solution. True.

    Correct Solution. False, since if $x = -1$, then it will satisfy $\frac{1}{x} < 2$, but not

$x > \frac{1}{2}$.

11. If $x < 2$, , then $x^2 < 4$.

    Wrong Solution. True.

    Correct Solution. False, since if $x = -3$, then it will satisfy $-3 < 2$, but $x^2 = (-3)^2 = 9$ is not less than 4.

12. $x^2 - 5x \leq 0$ is equivalent to $x - 5 \leq 0$.

    Wrong Solution. True, by dividing both sides of the inequality by x.

    Correct Solution. False, we cannot divide by x, since x may be a positive, a negative number, or zero.

13. If $\frac{-1}{2} \leq 2x \leq \frac{1}{2}$, then $-1 \leq x \leq 1$.

    Wrong Solution. True.

    Correct Solution. False, by dividing through by 2, we will get $\frac{-1}{4} \leq x \leq \frac{1}{4}$.

14. If $-1 \leq 3x - 2 \leq 1$, then $\frac{-1}{3} \leq x - 2 \leq \frac{1}{3}$.

    Wrong Solution. True.

    Correct Solution. False, by dividing through by 3, we will get $\frac{-1}{3} \leq x - \frac{2}{3} \leq \frac{1}{3}$.

II. Solutions with Wrong Steps:

Find the solution set of each of the following inequlaties:

1. $\frac{x+4}{x-1} < 0$.

    Wrong Solution. We need $x + 4 < 0$ and $x - 1 < 0$.

    Correct Solution. The sign diagram of $\frac{x+4}{x-1} < 0$ is

Therefore, the solution set is the interval $(-4, 1)$.

2. $\dfrac{x-4}{x+6} \leq 1.$

   Wrong Solution. We need $x - 4 \leq 1$ and $x + 6 \leq 1$.

   Correct Solution. We need to solve the inequality $\dfrac{x-4}{x+6} \leq 1$ as follows:

$$\begin{aligned} \frac{x-4}{x+6} - 1 &\leq 0 \\ \Rightarrow \frac{x-4-x-6}{x+6} &\leq 0 \\ \Rightarrow \frac{-10}{x+6} &\leq 0 \\ \Rightarrow \frac{10}{x+6} &\geq 0 \end{aligned}$$

   So $x + 6 > 0 \Rightarrow x > -6$. Therefore, the solution set is the interval $(-6, \infty)$.

Chapter 2

Functions And Graphs

Objectives

To introduce

- A two-dimensional coordinate system and graphs
- Functions, linear functions, and quadratic functions
- Properties of graphs
- Algebra of functions

2.1 A Two-dimensional Coordinate System and Graphs

Objectives

To introduce

- Cartesian Coordinate Systems
- The Distance and Midpoint Formulas
- Graphs and Equations
- Intercepts
- Circles, their Equations, and their Graphs

Solutions with Wrong Steps:

1. Find the midpoint of the line segment connecting the points whose coordinates are $P_1\left(-\frac{1}{3}, 2\right)$ and $P_2\left(0, -\frac{1}{2}\right)$.

   Wrong Solution. Midpoint $= \left(\frac{-1/3+0}{2}, \frac{2-1/2}{2}\right) = \left(-\frac{2}{3}, \frac{3/2}{2}\right) = \left(-\frac{2}{3}, 3\right)$.

   Correct Solution. Midpoint $= \left(\frac{-1/3+0}{2}, \frac{2-1/2}{2}\right) = \left(-\frac{1}{6}, \frac{3/2}{2}\right) = \left(-\frac{1}{6}, \frac{3}{4}\right)$.

2. Find the length of the line segment connecting the points $P_1(4, 6)$, and $P_2(11, -8)$.

   Wrong Solution. $d(P_1, P_2) = \sqrt{(11-4)^2 + (-8-6)^2} = \sqrt{(7)^2 + (14)^2} =$ $7 + 14 = 21$.

   Correct Solution. $d(P_1, P_2) = \sqrt{(11-4)^2 + (-8-6)^2} = \sqrt{(7)^2 + (14)^2} = \sqrt{(7)^2 + (2)^2(7)^2} = 7\sqrt{1+4} = 7\sqrt{5}$.

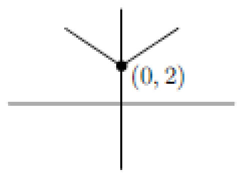
3. Find the distance between the point $P(x, 5x)$ and the origin where $x < 0$.

Wrong Solution. The origin $O(0,0) \Rightarrow d(P,O) = \sqrt{(x-0)^2+(5x-0)^2} = \sqrt{x^2+25x^2} = \sqrt{26x^2} = \sqrt{26}x$.

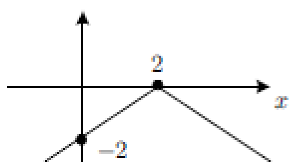
Correct Solution. $d(P,0) = \sqrt{(x-0)^2+(5x-0)^2} = \sqrt{x^2+25x^2} = \sqrt{26x^2} = \sqrt{26}|x|$. But $x<0 \Rightarrow d(P,0) = -\sqrt{26}x$.

4. Sketch the grpah of $y = -|x-2|$.

Wrong Solution. $y = -|x-2| \Rightarrow y = |-x+2| \Rightarrow y = |-x|+|2| = |x|+2$, and the graph is



Correct Solution. $y = -|x-2| \Rightarrow y \le 0$ and $y=0$ when $x=2$. Choose some points for $x<2$ and for $x>2$, for example, $(-2,-4),(-1,-3),(0,-2),(1,-1)$, $(2,0),(3,-1),\ldots$ etc. The graph is



5. Find the center and radius of the circle $2x^2+2y^2-4x+6x+1=0$.

Wrong Solution. $\Rightarrow x^2+y^2-4x+6x+1=0 \Rightarrow (x^2-4x)+(y^2+6x) = -1 \Rightarrow (x-2)^2+(y+3)^2 = -1+4+9 \Rightarrow (x-2)^2+(y+3)^2 = 12 \Rightarrow$ center $(2,-3)$, radius $= \sqrt{12}$.

Correct Solution. $\Rightarrow x^2+y^2-2x+3y+\frac{1}{2} = 0 \Rightarrow x^2-2x+y^2+3y = -\frac{1}{2} \Rightarrow (x-1)^2+\left(y+\frac{3}{2}\right)^2 = -\frac{1}{2}+1+\frac{9}{4} \Rightarrow (x-1)^2+\left(y+\frac{3}{2}\right)^2 = \frac{11}{4} \Rightarrow$ center is at

$\left(1, -\frac{3}{2}\right)$, and radius $= \frac{\sqrt{11}}{2}$.

6. Find the center of the circle $(x+2)^2 + (y-3)^2 = 5$.

   Wrong Solution. The center is $(2, -3)$.

   Correct Solution. The center is $(-2, 3)$.

7. Find the x-intercepts of the circle $(x+5)^2 + (y-2)^2 = 16$.

   Wrong Solution. x-intercepts $\Rightarrow (x+5)^2 = 16 \Rightarrow x+5 = 4$ or $x+5 = -4 \Rightarrow x = -1,\ x = -9 \Rightarrow (-1, 0)$ and $(-9, 0)$ are the x-intercepts.

   Correct Solution. x-intecepts $\Rightarrow$ Put $y = 0$, we get $(x+5)^2 + 4 = 16 \Rightarrow (x+5)^2 = 12 \Rightarrow x+5 = 2\sqrt{3}$ or $x+5 = -2\sqrt{3} \Rightarrow (2\sqrt{3} - 5, 0)$, and $(-2\sqrt{3} - 5, 0)$ are the x-intercepts.

2.2 Introduction to Functions

Objectives

To introduce

- Relations
- Functions
- Functional Notations
- How to Identify Functions
- The Greatest Integer Function

Solutions with Wrong Steps:

1. Let $f(x) = -3x^2 + 2x + 1$, evaluate $f(y+1)$.

   Wrong Solution. $f(y+1) = f(y) + f(1) \Rightarrow f(y+1) = (-3y^2 + 2y + 1) + (-3 + 2 + 1) = -3y^2 + 2y + 1$.

   Correct Solution. $f(y+1) = -3(y+1)^2 + 2(y+1) + 1 = -3(y^2 + 2y + 1) + 2y + 2 + 1 = -3y^2 - 6y - 3 + 2y + 3 = -3y^2 - 4y$.

2. Let $f(x) = \begin{cases} 3x - 2, & x < 1 \\ \dfrac{1}{x+1}, & x \geq 1 \end{cases}$, find $\frac{1}{6}f(2) + 3f\left(\frac{1}{2}\right)$.

   Wrong Solution. $\frac{1}{6}f(2) + 3f\left(\frac{1}{2}\right) = f\left(\frac{1}{3}\right) + f\left(\frac{3}{2}\right) = \left[3\left(\frac{1}{3}\right) - 2\right] + \frac{1}{\frac{3}{2} + 1} = -1 + \frac{2}{5} = -\frac{3}{5}$.

   Correct Solution. $\frac{1}{6}f(2) + 3f\left(\frac{1}{2}\right) = \frac{1}{6}\left(\frac{1}{2+1}\right) + 3\left(3\left(\frac{1}{2}\right) - 2\right) = \frac{1}{18} - \frac{3}{2} = -\frac{26}{18} = -\frac{13}{9}$.

3. Let $f(x) = [5x]$, where $[y]$ is the greatest integer $\leq y$, find $f\left(\frac{1}{3}\right)$.

Wrong Solution. $f(x) = [5x] = 5[x] \Rightarrow f\left(\frac{1}{3}\right) = 5\left[\frac{1}{3}\right] = (5)(0) = 0.$

Correct Solution. $f\left(\frac{1}{3}\right) = \left[\frac{5}{3}\right], \quad 1 \leq \frac{5}{3} < 2 \Rightarrow f\left(\frac{1}{3}\right) = 1.$

4. The statement "The relation $\{(-2,1),(-2,3),(4,3),(5,3)\}$ does not define a function" is true. Give a reason.

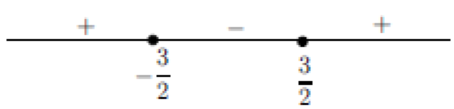
Wrong Solution. Because the ordered pairs $(-2,3),(4,3)$, and $(5,3)$ have the same second coordinate.

Correct Solution. Because the ordered pairs $(-2,1),(-2,3)$, have the same first coordinate.

5. Find the domain of the function $f(x) = \sqrt{4x^2 + 9}$.

Wrong Solution. We must have

$$4x^2 + 9 \geq 0 \Rightarrow (2x-3)(2x+3) \geq 0$$



$\Rightarrow$ Domain $= \left(-\infty, -\frac{3}{2}\right] \cup \left[\frac{3}{2}, \infty\right).$

Correct Solution. The expression $4x^2 + 9$ is positive for all real numbers x, thus the domain $= (-\infty, \infty)$.

6. Find the domain of the function $f(x) = \dfrac{\sqrt{x-1}}{x-1}$.

Wrong Solution. The domain $= (-\infty, 1) \cup (1, \infty)$.

Correct Solution. We must have $x - 1 \geq 0$ and $x \neq 1 \Rightarrow$ the domain $= (1, \infty)$.

7. The statement "The relation $y^2 = x$ is not a function by the vertical line test" is true. Give a reason.

<u>Wrong Solution</u>. Because the line $y = 4$ intersects the graph at two points $(-2, 4)$ and $(2, 4)$.

<u>Correct Solution</u>. Because the vertical line $x = 4$ intersects the graph at two points $(4, -2)$ and $(4, 2)$.

8. Determine whether or not 3 is in the range of $f(x) = \dfrac{2x-1}{x+1}$.

<u>Wrong Solution</u>. $f(3) = \dfrac{6-1}{3+1} = \dfrac{5}{4} \Rightarrow$ 3 is in the range of f.

<u>Correct Solution</u>. If $3 = \dfrac{2x-1}{x+1} \Rightarrow 3x+3 = 2x-1 \Rightarrow x = -4 \Rightarrow f(-4) = \dfrac{-8-1}{-4+1} = \dfrac{-9}{-3} = 3 \Rightarrow$. Thus 3 is in the range of f.

9. Determine whether the statement is True or False "If f is a function such that $f(a) = f(b)$, then $a = b$". Give a reason to your answer.

<u>Wrong Solution</u>. True: Because, let $f(x) = 3x + 2$, then $f(a) = f(b) \Rightarrow 3a + 2 = 3b + 2 \Rightarrow 3a = 3b \Rightarrow a = b$.

<u>Correct Solution</u>. False: For example, let $f(x) = x^2$, then $f(-2) = f(2)$ with $-2 \neq 2$.

2.3 Linear Functions

Objectives

To introduce

- Slopes of Lines
- Equation of a Line
- Parallel and Perpendicular Lines

Solutions with Wrong Steps:

1. Find the slope of the line passing through the points $(-2, 5)$, $(3, -4)$.

   Wrong Solution. The slope $= \frac{5+4}{3+2} = \frac{9}{5}$.

   Correct Solution. The slope $= \frac{5-(-4)}{-2-3} = \frac{9}{-5} = -\frac{9}{5}$.

2. Determine whether or not the lines $x = 4$ and $y = -2$ are perpendicular.

   Wrong Solution. The lines are not perpendicular since the slope of the line $x = 4$ is undefined, and the slope of the line $y = -2$ is 0.

   Correct Solution. $x = 4$ represents a vertical line, and the equation $y = -2$ represents a horizontal line $\Rightarrow$ they are perpendicular lines.

3. Find the equation of the line with slope $-\frac{1}{2}$ that passes through $(-2, 5)$.

   Wrong Solution. $(y-5) = -\frac{1}{2}(x+2) \Rightarrow -2y - 5 = x + 2 \Rightarrow x + 2y + 7 = 0$.

   Correct Solution. $y - 5 = -\frac{1}{2}(x+2) \Rightarrow 2y - 10 = -x - 2 \Rightarrow x + 2y - 8 = 0$.

4. Determine whether the statement is True or False "The graph of the line $3x + 5y + 6 = 0$ is increasing". Give a reason.

Wrong Solution. True. Because the line is of a positive slope $\frac{3}{5}$.

Correct Solution. False. $3x + 5y + 6 = 0 \Rightarrow 5y = -3x - 6 \Rightarrow y = -\frac{3}{5}x - \frac{6}{5} \Rightarrow$ the slope $= -\frac{3}{5} \Rightarrow$ the slope of the line is negative $\Rightarrow$ decreasing.

2.4 Quadratic Functions

Objectives

To introduce

- Quadratic Functions
- Upward or Downward Parabolas
- Vertex of a Parabola
- Axis of Symmetry
- Maximum and Minimum of a Quadratic Function

True or False Statements (Give a Reason):

1. $f(x) = ax^2 + bx + c$ is a quadratic function for any values a, b, and c.

   Wrong Answer. True.

   Correct Answer. False, since if $a = 0$, then $f(x)$ is not a quadratic function.

2. $f(x) = ax^2 + bx + c$ is the standard form of a quadratic function.

   Wrong Answer. True.

   Correct Answer. False, since $f(x) = a(x-h)^2 + k$ is the standard form of a quaratic function.

3. The axis of symmetry of $f(x) = a(x-1)^2 + 3$ is $y = 3$.

   Wrong Answer. True.

   Correct Answer. False, since the axis of symmetry is $x = 1$.

4. $f(x) = -2(x-1)^2 + 3$ is a parabola opens upward.

Wrong Answer. True, since $k = 3 > 0$.

Correct Answer. False, since $a = -2 < 0$, therefore $f(x)$ is a parabola opens downward.

5. The vertex of the parabola $f(x) = 3x^2 + 6x$ is $(-2, 0)$.

Wrong Answer. True, since $h = \frac{-b}{a} = \frac{-6}{3} = -2$ and so $k = f(h) = f(-2) = 12 - 12 = 0$.

Correct Answer. False, since $h = \frac{-b}{2a} = \frac{-6}{6} = -1$ and so $k = f(h) = f(-1) = 3 - 6 = -3$. So, the vertex is $(-1, -3)$.

6. The maximum of $f(x) = -2(x-1)^2 + 3$ is 1.

Wrong Answer. True, since $h = 1$.

Correct Answer. False, since the maximum is $k = f(h) = f(1) = 3$.

7. The vertex of the parabola $f(x) = 2x^2 - 2x$ is $(-2, 12)$.

Wrong Answer. True, since $h = -2$.

Correct Answer. False, since $h = \frac{-b}{2a} = \frac{2}{4} = \frac{1}{2}$ and so $k = f\left(\frac{1}{2}\right) = \frac{-1}{2}$, i.e., the vertex is $\left(\frac{1}{2}, \frac{-1}{2}\right)$.

8. The vertex of the parabola $f(x) = -2(x-1)^2 + 3$ is $(3, 1)$.

Wrong Answer. True.

Correct Answer. False, since $h = 1, k = 3$ and so the vertex is $(h, k) = (1, 3)$ and not $(3, 1)$.

9. The axis of symmetry of the parabola $f(x) = a(x-1)^2 + 3$ is $y = 1$.

Wrong Answer. True, since $h = 1$.

Correct Answer. False, since $h = 1$ and so $x = 1$ is the axis of symmetry.

10. Every quadratic function intersects the x-axis at least at one point.

    Wrong Answer. True.

    Correct Answer. False, since $f(x) = x^2 + 2$ does not intersect the x-axis at any point.

11. The minimum of $f(x) = 3x^2 + 6x$ is 0.

    Wrong Answer. True, since $c = 0$ and so $k = 0$.

    Correct Answer. False, since $h = \frac{-b}{2a} = -1$ and so $k = -3$. So, the minimum is $f(-1) = -3$.

12. The maximum of $f(x) = 3x^2 + 6x$ is -3.

    Wrong Answer. True, since $k = -3$.

    Correct Answer. False, since $a = 3 > 0$, therefore the parabola opens upward and so -3 is a minimum and there is no maximum value for f (see (11)).

13. Every quadratic function has a maximum and a minimum.

    Wrong Answer. True.

    Correct Answer. False, since $f(x) = x^2$ has a minimum but no maximum.

II. Solutions with Wrong Steps:

Use the technique of completing the square to find the standard form of each of the following quadratic functions:

1. $f(x) = 2x^2 - 12x + 19$.

Wrong Answer. We will add and subtract half the coefficient of x squared to get $f(x) = 2x^2 - 12x + 36 + 19 - 36, \ldots$, etc.

Correct Answer. We need first to factor 2 to get $f(x) = 2(x^2 - 6x) + 19$, then we add and subtract half the coefficient of x squared to get $f(x) = 2(x^2 - 6x + 9 - 9) + 19 = 2(x - 3)^2 + 1$.

2. $f(x) = 2(x^2 - 6x) + 19$.

   Wrong Answer. After we add and subtract 9, we get

$$f(x) = 2(x^2 - 6x + 9 - 9) + 19$$
$$= 2(x^2 - 6x + 9) + 19 - 9 = 2(x - 3)^2 + 10.$$

   Correct Answer. After we add and subtract 9, we need not to forget to multiply 9 by 2 as follows:

$$f(x) = 2(x^2 - 6x + 9 - 9) + 19$$
$$= 2(x^2 - 6x + 9) - 2(9) + 19 = 2(x - 3)^2 + 1.$$

3. $f(x) = 2(x^2 - 6x) + 19$.

   Wrong Answer. We add 9 to complete the square to get $2(x^2 - 6x + 9) + 19$.

   Correct Answer. We need to add and subtract 9 as explained in (2).

4. $f(x) = 2(x^2 - 6x) + 19$.

   Wrong Answer. We add and subtract 6 which is $(2)\left(\frac{6}{2}\right)$.

   Correct Answer. We need to add and subtract 9 as explained in (2).

5. $f(x) = -2(x^2 - 6x) + 19$.

   Wrong Answer. We add and subtract 9 to get $f(x) = -2(x^2 - 6x + 9 - 9) + 19$, then $f(x) = -2(x^2 - 6x + 9) - 2(9) + 19$.

Correct Answer. $f(x) = -2(x^2 - 6x + 9 - 9) + 19$, then $f(x) = -2(x^2 - 6x + 9) - 2(-9) + 19 = -2(x-3)^2 + 37$.

2.5 Properties of Graphs

Objectives

To introduce

- Symmetry
- Even and Odd Functions
- Transformation of Graphs
- Reflection of Graphs
- Compressing and Stretching of Graphs

True or False Statements (Give a Reason):

1. The graph of $y = x^2$ is symmetric with respect to the x-axis.

   Wrong Answer. True, since if we replace x by $-x$, we will get $y = (-x)^2 = x^2$.

   Correct Answer. False, since if we replace x by $-x$ and we get the same equation, then the graph of that equation is symmetric with respect to the y-axis and not the x-axis.

2. The graph of $y^2 = x$ is symmetric with respect to the y-axis.

   Wrong Answer. True, since if we replace y by $-y$, we will get $(-y)^2 = y^2 = x$.

   Correct Answer. False, since if we replace y by $-y$ and we get the same equation, then the graph of that equation is symmetric with respect to the x-axis and not the y-axis.

3. If the graph of an equation is symmetric with respect to the origin, then it is symmetric with respect to the x-axis and the y-axis.

Wrong Answer. True, since if the graph is symmetric with respect to the origin, then that means replacing x by $-x$ and replacing y by $-y$ will give you the same equation which is the definition of the symmetry with respect to the x-axis and the y-axis, respectively.

Correct Answer. False, since replacing x by $-x$ and y by $-y$ has to be done at the same time. For example, $y = x^3$ is symmetric with respect to the origin, but not symmetric with respect to the x-axis or the y-axis.

4. The graph of $y = x^3 - x$ is not symmetric with respect to the origin.

Wrong Answer. True, since if we replace x by $-x$ and y by $-y$, we will get $-y = (-x)^3 - (-x)$ and that implies $-y = -x^3 + x$ and so $y = x^3 + x$ which is different than $y = x^3 - x$.

Correct Answer. False, since if we do the same thing as above, we will get

$$-y = (-x)^3 - (-x)$$
$$-y = -x^3 + x$$
$$y = -(-x^3 + x) = -(-x^3) - (x) \Rightarrow$$
$$y = x^3 - x$$

which means the graph is symmetric with respect to the origin.

5. $f(x) = x^3 + 1$ is an odd function.

Wrong Answer. True, since $f(x)$ is of degree 3 which is an odd number.

Correct Answer. False, since if $f(-x) = (-x)^3+1 = -x^3+1$ where $-f(x) = -x^3-1$ and so $f(-x) \neq -f(x)$, i.e., $f(x)$ is not an odd function.

6. $f(x) = x^4 + x^2 + x$ is an even function.

Wrong Answer. True, since $f(x)$ is of degree 4 which is an even number.

Correct Answer. False, since $f(-x) = (-x)^4 + (-x)^2 + (-x) = x^4 + x^2 - x$ and so $f(-x) \neq f(x)$, i.e., $f(x)$ is not an even function.

7. The graph of $y - 1 = x^2$ is the graph of $y = x^2$ shifted down vertically 1 unit.

   Wrong Answer. True, since c in the definition is -1 and so the translation is down.

   Correct Answer. False, since $y - 1 = x^2$ is the same as $y = x^2 + 1$ and now c in the definition is 1 and so the translation is up and not down. In other words, the definition is to be applied to $y = f(x) + c$.

8. The graph of $f(-x+1)$ is the graph of $y = f(-x)$ shifted right horizontally 1 unit.

   Wrong Answer. True, since if we solve the equation $-x + 1 = 0$, we will get $x = 1 > 0$ and so the translation is one unit to the right.

   Correct Answer. False, since if we compare $f(-x+1)$ to $f(-x)$ according to the definition, then the translation is one unit to the left.

9. The graph of $x = (y-1)^3$ is the graph of $x = y^3$ shifted right horizontally 1 unit.

   Wrong Answer. True.

   Correct Answer. False, since the change is in y, therefore the shift is either up or down and so the shift is 1 unit upward.

2.6 The Algebra of Functions

Objectives

To introduce

- Operations on Functions
- The Difference Quotient
- Composition of Functions

True or False Statements (Give a Reason):

1. If $f(x) = x^2 + 3$, then $f(x+h) = x^2 + h^2 + 3$.

   Wrong Answer. True, since $f(x+h) = (x+h)^2 + 3 = x^2 + h^2 + 3$.

   Correct Answer. False, since $f(x+h) = (x+h)^2 + 3 = x^2 + 2xh + h^2 + 3$.

2. $(fg)(x) = f(g(x))$ for any functions f and g.

   Wrong Answer. True.

   Correct Answer. False, since $(fg)(x) = f(x) \cdot g(x)$ which is different than $f(g(x))$. For example, if $f(x) = x$ and $g(x) = x+1$, then $f(g(x)) = x+1$ while $(fg)(x) = x^2+x$.

3. If $f(x) = x$ and $g(x) = x$, then the domain of $\left(\frac{f}{g}\right)(x)$ is $(-\infty, \infty)$.

   Wrong Answer. True, since $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = 1$.

   Correct Answer. False, since the domain of $\frac{f}{g}$ consists of all real numbers formed by the intersection of the domain of f and the domain of g with $g(x) \neq 0$. So, the domain of $\frac{f}{g}$ is $(-\infty, 0) \cup (0, \infty)$.

4. The domain of $f+g$ consists of all real numbers formed by the union of the domain of f and the domain of g.

Wrong Answer. True.

Correct Answer. False, since the domain of $f+g$ consists of all real numbers formed by the intersection of the domain of f and the domain of g.

5. $(f \circ g)(x) = f(x) \cdot g(x)$ for any funcions f and g.

Wrong Answer. True.

Correct Answer. False, see (2).

6. $(f \circ g)(x) = (g \circ f)(x)$ for any funcions f and g.

Wrong Answer. True.

Correct Answer. False, since if $f(x) = x^2$ and $g(x) = x + 1$, then $f(g(x)) = (x+1)^2 = x^2 + 2x + 1$ while $g(f(x)) = x^2 + 1$.

Chapter 3

Polynomials And Rational Functions

Objectives

To introduce

- The Remainder Theorem and The Factor Theorem
- Polynomial Functions of Higher Degree
- Zeros of Polynomial Functions
- The Fundamental Theorem of Algebra
- Graphs of Rational Functions

3.1 The Remainder Theorem and The Factor Theorem

Objectives

To introduce:

- Division of Polynomials
- Synthetic Division
- The Remainder and Factor Theorems

I. Solutions with Wrong Steps:

1. Divide $-2x^3 - 3x^2 - 5x + 7$ by $2 - x$.

<u>Wrong Solution</u>.

$$
\begin{array}{r|l}
 & -\ x^3 - \frac{1}{2}x^4 \\
\hline
2-x & -2x^3 - 3x^2 - 5x + 7 \\
 & -2x^3 + x^4 \\
\hline
 & -\ x^4 - 3x^2 - 5x + 7 \\
 & -\ x^4 + \frac{1}{2}x^5 \\
\hline
 & -\frac{1}{2}x^5 - 3x^2 - 5x + 7
\end{array}
$$

and the student does not know how to stop!!

<u>Correct Solution</u>.

$$
\begin{array}{r|l}
 & 2x^2 + 7x + 19 \\
\hline
-x+2 & -2x^3 - 3x^2 - 5x + 7 \\
 & -2x^3 + 4x^2 \\
\hline
 & \quad -7x^2 - 5x + 7 \\
 & \quad -7x^2 + 14x \\
\hline
 & \qquad -19x + 7 \\
 & \qquad -19x + 38 \\
\hline
 & \qquad\qquad -31
\end{array}
$$

$\Rightarrow$ The quotient $= 2x^2 + 7x + 19$.

The remainder $= -31$.

2. Use synthetic division to divide

$$-3x^4 + 2x^3 + 7x^2 - 4 \text{ by } x + 1.$$

<u>Wrong Solution</u>.

$$\begin{array}{r|rrrrr} -1 & -3 & 2 & 7 & 0 & -4 \\ & & 3 & 1 & -6 & -6 \\ \hline & -3 & -1 & 6 & 6 & 2 \end{array}$$

$\Rightarrow$ The quotient $= -3x^3 - x^2 + 6x + 6$.

The remainder $= 2$.

<u>Correct Solution</u>.

$$\begin{array}{r|rrrrr} -1 & -3 & 2 & 7 & 0 & -4 \\ & & 3 & -5 & -2 & 2 \\ \hline & -3 & 5 & 2 & -2 & -2 \end{array}$$

$\Rightarrow$ The quotient $= -3x^3 + 5x^2 + 2x - 2$.

The remainder $= -2$.

3. Use synthetic division to divide

$$2x^4 + 3x + 4 \text{ by } x + 1.$$

<u>Wrong Solution</u>.

$$\begin{array}{r|rrr} -1 & 2 & 3 & 4 \\ & & -2 & -1 \\ \hline & 2 & 1 & 3 \end{array}$$

$\Rightarrow$ The quotient $= 2x^3 + x^2$.

The remainder $= 3$.

<u>Correct Solution</u>.

$$\begin{array}{r|rrrrr} -1 & 2 & 0 & 0 & 3 & 4 \\ & & -2 & 2 & -2 & -1 \\ \hline & 2 & -2 & 2 & 1 & 3 \end{array}$$

$\Rightarrow$ The quotient $= 2x^3 - 2x^2 + 2x + 1$.

The remainder $= 3$.

4. Let $P(x) = -5x^3 + 2x^2 - x + 7$. Use the Remainder Theorem to find $P(2)$.

<u>Wrong Solution</u>. $P(2) = -40 + 8 - 2 + 7 = -27$.

<u>Correct Solution</u>. The above solution gives the value of P at $x = 2$. But to apply the Remainder Theorem, one needs to get $P(2)$ as a remainder of a division as follows:

$$\begin{array}{r|rrrr} 2 & -5 & 2 & -1 & 7 \\ & & -10 & -16 & -34 \\ \hline & -5 & -8 & -17 & -27 \end{array}$$

$\Rightarrow \quad P(2) =$ The remainder $= -27$.

5. Use synthetic division to find the quotient and remainder when dividing

   $-4x^3 + 2x^2 + 4x - 1$ by $2x - 1$.

   Wrong Solution. Write $2x - 1$ as $2\left(x - \frac{1}{2}\right)$

$$\Rightarrow \quad \begin{array}{r|rrrr} 1/2 & -4 & 2 & 4 & -1 \\ & & -2 & 0 & 2 \\ \hline & -4 & 0 & 4 & 1 \end{array}$$

   $\Rightarrow$ The quotient $= -4x^2 + 4$.

   The remainder $= 1$.

   Correct Solution. The remainder in the above solution is correct since it is equal to $P\left(\frac{1}{2}\right)$ according to the Remainder Theorem. While the quotient must be divided by 2 to get the quotient $= -2x^2 + 2$.

3.2 Polynomial Functions of Higher Degree

Objectives

- To Investigate the Far-Left and Far-Right Behavior of the Graph of a Polynomial
- To discuss the Possibility of the Existence of Maximum and Minimum Values of a Polynomial
- To Find the Real Zeros of a Polynomial
- To Discuss Zero of Multiplicity n
- To Introduce a Graphing Procedure for Graphing Polynomial Function

I. **True or False Statement (Give a Reason):**

1. Let $P(x)$ be a polynomial such that $P(0) > 0$ and $P(1) > 0$, then $P(x)$ has no real zero in the interval $(0, 1)$.

   Wrong Answer. True.

   Correct Answer. False. Consider the polynomial $P(x) = (3x - 1)(2x - 1) \Rightarrow P(0) = 1 > 0$ and $P(1) = (2)(1) = 2 > 0$ while $P(x)$ has two zeros $\frac{1}{3}$ and $\frac{1}{2}$ in $(0, 1)$.

2. The graph of the polnomial $P(x) = (x + 1)(x - 3)^6$ crosses the x-axis at $x = 3$.

   Wrong Answer. True. Because 3 is a zero of $P(x)$.

   Correct Answer. False. 3 is a zero of multiplicity 6 which is an even number $\Rightarrow$ the graph of P touches the x-axis but does not cross it.

II. **Solutions with Wrong Steps**:

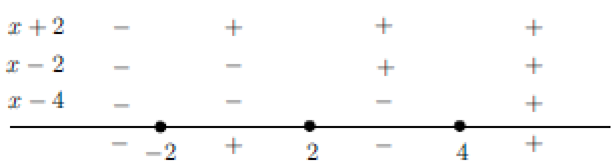
1. Examine the leading term and determine the far-left and far-right behavior of the graph of the polynomial $P(x) = -4x^3 + 3x^2 - 5x + 1$.

   Wrong Solution. The leading coefficient $= -4 < 0 \Rightarrow$ the graph goes down to its far left and down to its far right.

   Correct Solution. The leading coefficient $= -4 < 0$ and the polynomial is of odd degree $\Rightarrow$ the graph goes up to its far left and down to its far right.

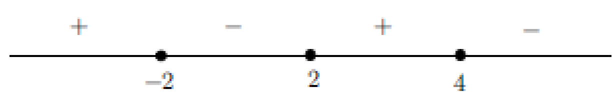
2. Determine the intervals on which the polynomial $P(x) = -(x+2)(x-2)(x-4)$ has (if any) a relative maximum or a relative minimum value.

   Wrong Solution. The zeros of $P(x)$ are $-2, 2$, and $4 \Rightarrow$ the sign diagram of $P(x)$ is



   $\Rightarrow$ a relative maximum value in the interval $(-2, 2)$ and a relative minimum value in the interval $(2, 4)$.

   Correct Solution. The above solution is wrong because the student forgot the negative sign preceding the factors $(x+2)(x-2)(x-4) \Rightarrow$ The correct sign diagram is



   $\Rightarrow$ a relative minimum value in the interval $(-2, 2)$ and a relative maximum value in the interval $(2, 4)$.

3.3 Zeros of Polynomial Functions

Objectives

To introduce

- The Rational Zero Theorem
- Upper and Lower Bounds for Real Zeros
- Descartes' Rule of Signs
- Zeros of a Polynomial Function

True or False Statements (Give a Reason):

1. The list of all rational zeros of the polynomial

$$P(x) = 4x^3 - 10x^2 + 11x - 12$$

contains 36 rational numbers.

Wrong Answer. True. Since a rational zero is of the form $\frac{p}{q}$ where

$$p : \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12 \quad \text{and } q : \pm 1, \pm 2, \pm 4$$

which gives 36 possibilities.

Correct Answer. False. $\frac{p}{q}$ must be in reduced form, thus the list of all possible $\frac{p}{q}$ is

$$\pm 1, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm 2, \pm 3, \pm \frac{3}{2}, \pm \frac{3}{4}, \pm 4, \pm 6, \pm 12,$$

i.e., 20 possibilities and not 36.

2. According to the Rational Zero Theorem, the polynomial $P(x) = x^2 + 1$ has two rational zeros (Give a reason).

Wrong Answer. True. $\quad p: \pm 1, \quad q: \pm 1 \Rightarrow$ all $\frac{p}{q}$ are $\pm 1 \Rightarrow P$ has two rational zeros.

Correct Answer. False. ± 1 is the list of all possible real zeros but neither 1 nor -1 is a zero of P since $P(1) \neq 0$ and $P(-1) \neq 0$.

3. If $P(x)$ is a polynomial with real positive coefficinets, then the real zeros of P, if they exist, are negative real numbers.

   Wrong Answer. True. $\quad$ For example, $P(x) = x + 1$ has the zero -1.

   Correct Answer. False. For example, $P(x) = x^2 + x$ has the zeros 0 and -1.

4. The number 2 is the only upper bound of the positive zeros of the polynomial $P(x) = 2x^3 + 7x^2 - 4x - 14$.

   Wrong Answer. True. $\quad$ Since

   $$\begin{array}{r|rrrr} 2 & 2 & 7 & -4 & 14 \\ & & 4 & 22 & 36 \\ \hline & 2 & 11 & 18 & 12 \end{array}$$

   Correct Answer. False. 2 is one of the upper bounds of the positive zeros of $P(x)$. As a matter of fact, any number greater than 2 can also be considered as an upper bound.

5. According to Descrates' Rule of Signs, the polynomial $P(x) = 3x^3 - 2x^2 + x + 11$ has two positive real zeros.

   Wrong Answer. True. $\quad$ Since $P(x) = 3x^3 - 2x^3 + x + 11$ has two variations in sign.

   Correct Answer. False. $\quad P(x)$ has two variations in sign, thus $P(x)$ will have either two positive real zeros or no positive real zeros.

6. If $P(x)$ is a polynomial with integer coefficients and with no rational zeros, then the zeros of $P(x)$ are irrational.

   Wrong Answer. True. $\quad$ For example, $P(x) = x^2 - 2$ has no rational zeros and has two irrational zeros $\pm\sqrt{2}$.

Correct Answer. False. For example, $P(x) = x^2 + 1$ has neither rational zeros nor irrational zeros. The zeros are $\pm i$.

3.4 The Fundamental Theorem of Algebra

Objectives

To introduce

- The Fundamental Theorem of Algebra
- The number of Zeros of a Polynomial Function
- The Conjugate Pair Theorem
- How to Find a Polynomial Function with Given Zeros

True or False Statements (Give a Reason):

1. Every polonomial function has at least one real zero.

   Wrong Answer. True, because of the Fundamental Theorem of Algebra.

   Correct Answer. False, because the theorem says that every polynomial function has at least one complex zero. For example, $P(x) = x^2 + 1$ has no real zeros but has two nonreal zeros $\pm i$.

2. The graph of every polynomial function has at least one x-intercept.

   Wrong Answer. True, because of the Fundamental Theorem of Algbera.

   Correct Answer. False, since the graph of $P(x) = x^2 + 1$ does not intersect the x-axis.

3. If $a + bi$ is a zero of a polynomial $P(x)$, then $a - bi$ is also a zero of $P(x)$.

   Wrong Answer. True, because of the Conjugate Pair Theorem.

   Correct Answer. False, because the Conjugate Pair Theorem is true if $P(x)$ is a polynomial with real coefficients. For example, $P(x) = x - i$ is a polynomial where i is a zero but $-i$ is not a zero.

4. The complex zeros of a polynomial function always occur in conjugate pairs.

   Wrong Answer. True, because of the Conjugate Pair Theorem.

   Correct Answer. False, see (4).

5. If two polynomials have exactly the same zeros, then their graphs are identical.

   Wrong Answer. True.

   Correct Answer. False, because the graph of $f(x) = x^2$ and $g(x) = 2x^2$ are not identical while both have the same zeros.

3.5 Graphs of Rational Functions and Their Applications

Objectives

To introduce

- Vertical and Horizontal Asymptotes
- A Sign Property of Rational Functions
- A General Graphing Procedure
- Slant Asymptotes
- Graph Rational Functions that have a Common Factor

True or False Statements (Give a Reason):

1. Every rational function has a vertical asymptote.

   Wrong Answer. True.

   Correct Answer. False, since $f(x) = \dfrac{x}{x^2+1}$ has no vertical asymptote.

2. Every rational function has a horizontal asymptote.

   Wrong Answer. True.

   Correct Answer. False, since $f(x) = \dfrac{x^2}{x+1}$ is a rational function with no horizontal asymptote.

3. Every rational fnction has at most one vertical asymptote.

   Wrong Answer. True.

Correct Answer. False, since $f(x) = \dfrac{x}{x^2 - 1}$ has two vertical asymptotes, $x = -1$ and $x = 1$.

4. Every rational function has a slant asymptote.

Wrong Answer. True.

Correct Answer. False, since $f(x) = \dfrac{x+2}{x+1}$ has no slant asymptote.

5. The rational function $f(x) = \dfrac{x^2 - 1}{x+1}$ has one vertical asymptote.

Wrong Answer. True, since $x = -1$ is a vertical asymptote.

Correct Answer. False, since $f(x) = \dfrac{x^2-1}{x+1} = \dfrac{(x-1)(x+1)}{x+1} = x - 1,\ x \neq -1$. So, $f(x)$ has no vertical asymptotes. The graph has an open circle at $(-1, -2)$.

6. The rational function $f(x) = \dfrac{x - x^2}{x + x^2 + 1}$ has $y = 1$ as a horizontal asymptote.

Wrong Answer. True.

Correct Answer. False, since $f(x) = \dfrac{x - x^2}{x + x^2 + 1} = \dfrac{-x^2 + x}{x^2 + x + 1}$ and so $y = -1$ is the horizontal asymptote.

7. The graph of the rational function does not intersect its horizontal asymptote.

Wrong Answer. True.

Correct Answer. False, since for example $f(x) = \dfrac{x^2+1}{x^2+x}$ has $y = 1$ as a horizontal asymptote and it intersects the graph at the point $(1, 1)$.

8. $f(x) = \dfrac{\sqrt{x} - 2}{x + 4}$ is a rational function.

Wrong Answer. True.

Correct Answer. False, since $g(x) = \sqrt{x} - 2$ is not a polynomial.

4.1 Inverse Functions

Objectives

To introduce

- Inverse Functions
- Graphs of Inverse Functions
- Composition of a Function and its Inverse
- How to Find an Inverse Function.

True or False Statements (Give a Reason):

1. If $f(x) = x + 1$, then $f^{-1}(x) = \dfrac{1}{x+1}$.

   Wrong Answer. True, since $f^{-1}(x) = \dfrac{1}{f(x)}$.

   Correct Answer. False, since $f^{-1}(x)$ is different than the reciprocal of the function and $f^{-1}(x)$ in this case is $f^{-1}(x) = x - 1$.

2. The function $f(x) = x^2$ has an inverse.

   Wrong Answer. True, since f is a polynomial and a known function.

   Correct Answer. False, since $f(x) = x^2$ is not a one-to-one function, therefore has no inverse.

3. Every function has an inverse function.

   Wrong Answer. True.

   Correct Answer. False, see (2), for example.

4. If $(2, 3)$ is a point on the graph of a one-to-one function f, then $\left(\frac{1}{2}, \frac{1}{3}\right)$ is a point on the graph of f^{-1}.

Wrong Answer. True, since $f^{-1}(x) = \frac{1}{f(x)}$.

Correct Answer. False, since $(2, 3)$ is on the graph of f, therefore $(3, 2)$ is on the graph of f^{-1}.

5. If $(f \circ g)(a) = a$ and $(g \circ f)(a) = a$ for some constant a, then f and g are inverse functions to each other.

Wrong Answer. True, since these are the conditions for the inverse functions.

Correct Answer. False, since the conditions for the inverse functions are $(f \circ g)(a) = a$ and $(g \circ f)(a) = a$ for all a in the domain of $g = f^{-1}$, and f, respectively, provided that f is a one-to-one function.

6. The graph of a function f and its inverse f^{-1} do not intersect.

Wrong Answer. True, since the graphs of f and f^{-1} are symmetric to each other with respect to $y = x$.

Correct Answer. False, since one can easily check that the functions $f(x) = x^3$ and $g(x) = \sqrt[3]{x}$ are inverses to each other and their graphs intersect at $(1, 1)$ and $(-1, -1)$.