Reading Mathematical Expressions

Expression	Reads	Note
$a + b = c$	a plus b is equal to c	Addition; c is the sum
$a - b = c$	a minus b equals c	Subtraction; c is the difference
$a \cdot b = c$	a times b is equal to c	Multiplication; c is the product
$a \div b = c$	a divided by b equals c	Division; c is the quotient
$\frac{a}{b}$ or a/b	a over b or a by b	Fraction; a is the numerator, b is the denominator
$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$	one half, one third, one fourth	Reciprocals of 2 , 3 and 4
$\frac{5}{2}, \frac{2}{3}, \frac{7}{10}$	five halves, two thirds, seven tenths	
a^b	a to the b; a to the bth power	a: base , b: exponent
a^2, a^3, a^{-1}	a squared, a cubed, a inverse	
$\sqrt[n]{a}$	the nth root of a	nth radical a : radicand n : index
$\sqrt{a}$, $\sqrt[3]{a}$	square root of a, cube root of a	
$a < b$	a is less than b	Inequalities
$a \le b$	a is less than or equal to b	
$a > b$	a is greater than b	
$a \ge b$	a is greater than or equal to b	
$x \in A$	x belongs to A or x is in A	Between an element and a set
$A \subset B$	A is a subset of B	Between two sets
ϕ	the empty (null) set	$\phi \ne \{\phi\}$
$A \cup B$	A union B	Between two sets
$A \cap B$	A intersection B	Between two sets

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1. If $x = \frac{1}{9}$, $y = -5$ and $w = -\frac{5}{7}$, then find

a) $x + y$

b) $\frac{x}{y}$

c) $2x^2$

d) $\frac{x+w}{w-x}$

e) $7\frac{1}{5} - 4\frac{1}{8} \div 1\frac{1}{4}$

2. Find the exact value of

a) $1.32 + 0.132$

b) $1.05 - 100.3$

c) $26.06 \div 25$

d) $1.5 \div 0.15$

e) $\frac{1.2 \times 1.04}{0.06}$

3. a) Which is larger π or $\frac{22}{7}$? ($\pi \approx 3.14159$).

b) Which is smaller $\frac{3}{5}$ or $\frac{1}{2}$?

c) Express $\frac{1}{3}$ in the decimal form.

d) Express $0.\overline{4}$ as a fraction.

e) Express 0.62 as a fraction in its lowest terms.

f) Find the reciprocal of the mixed number $-2\frac{3}{5}$.

g) (i) $\sqrt{196} =$ **(ii)** $\sqrt{30.25} =$ **(iii)** $\sqrt{0.25 - 0.16} =$

P.2 Recitation Exercises

Which one of the following statements is TRUE?

A) Every rational number has a multiplicative inverse.

B) Every even integer has an additive inverse.

C) $\pi = \frac{22}{7}$

D) The sum of two irrational numbers is always irrational.

E) If x is any integer and y is any irrational number, then $\frac{x}{y}$ is irrational.

F) The distributive law states that $a \div (b + c) = (a \div b) + (a \div c)$.

G) Any irrational number has a terminating or repeating decimal expansion.

1. Given the sets

   $A = \{-4, -2, 0, 4, 6, 8, 9\}, \quad B = \{y \mid y \text{ is an even prime number}\},$

   $C = \{y \mid y \text{ is a composite number less than } 9\}$. Then $A \cap (B \cup C) =$

   A) $A \cup B$

   B) $B \cap C$

   C) C

   D) A

   E) $\emptyset$

2. Given $\frac{1}{3} \le x < \frac{2}{3}$, the expression $\left|x - \frac{2}{3}\right| - \left|\frac{1}{4} - x\right|$ simplifies to

   A) $-\frac{11}{12}$

   B) $2x - \frac{11}{12}$

   C) $\frac{11}{12} - 2x$

   D) $-\frac{5}{12}$

   E) $\frac{5}{12}$

3. If $A = \{x|\, x \leq -3\} \cup \{x|x > 1\}$, $B = [-6, 8)$ and $C = \{x|1 \geq x > -1\}$, then find

a) $A \cap B$

b) $(A \cap B) \cup C$

4. Writing an Algebraic Model for each of the following:

a) The maximum recommended heart rate M for adults is the difference of 220 and the person's age A. Write a model to represent an adult's maximum recommended heart rate in terms of age.

b) After eating at a restaurant, it is customary to leave a tip T for the server for at least 15% of the cost of the meal C. Write a model to represent the amount of the tip based on the cost of the meal.

5. The post office will accept only packages for which the length plus the "girth" (distance around) is no more than 108 in. Thus for the package in the figure, we must have

$$L + 2(x + y) \leq 108$$

a) Will the post office accept a package that is 6 in. wide, 8 in. deep, and 5 ft long? What about a package that measures 2 ft by 2 ft by 4 ft ?

b) What is the greatest acceptable length for a package that has a square base measuring 9 in. By 9 in. ?
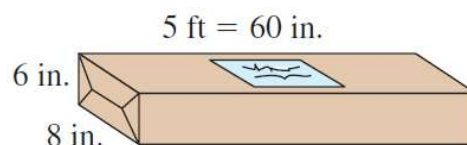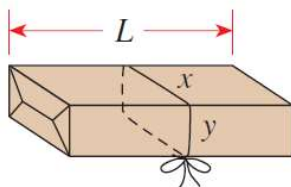
1. Write the number indicated in each statement in scientific notation:

   a) A light-year, the distance that light travels in one year, is about 5,900,000,000,000 mi.

   b) The diameter of an electron is about 0.0000000000004 cm.

   c) The distance from the earth to the sun is about 93 million miles.

   d) The mass of the earth is about 5,970,000,000,000,000,000,000,000 kg.

2. The expression $\left(\dfrac{xy^{-2}z^{-3}}{x^2y^3z^{-4}}\right)^{-3}$ is equal to

   A) $\dfrac{x^3y^{15}}{z^3}$

   B) $\dfrac{x^3y^3}{z^3}$

   C) $\dfrac{z^3y^{15}}{x^9}$

   D) $\dfrac{x^9y^3}{z^6}$

   E) $\dfrac{z}{xy}$

3. If $M = 2^{-1} + 3^{-1}$ and $N = 36 \times 10^{-4}$, then find the exact value of MN.

4. Which one of the following statements is TRUE?

   A) $\left(\dfrac{2}{3}\right)^{-2} = -\dfrac{4}{9}$

   B) $(-5)^4 = -5^4$

   C) $(x^2)^3 = x^5$

   D) $(2x^4)^3 = 2x^{12}$

   E) $\left(\dfrac{1}{2}\right)^{-1} = \dfrac{1}{2^{-1}}$

5. Explain why scientific notation used.

P.4 Recitation Exercises

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1. Simplify the expression $\left(\frac{-27}{8}\right)^{-\frac{2}{3}} - (2)^{\frac{1}{6}} \left(-32^{\frac{1}{6}}\right) + 3(-2)^0$.

2. Simplify the expression $\left(\frac{x^{\frac{1}{2}} y^2}{2 \, y^{\frac{1}{4}}}\right)^4 \left(\frac{4x^{-2}y^{-4}}{y^2}\right)^{\frac{1}{2}}$, where $x > 0$ and $y > 0$.

3. If $x < -3$, then simplify the expression $\sqrt{(-7)^2} - \sqrt{(3x + 7)^2} + \sqrt[3]{(-7)^3}$ and write the result without absolute value symbols.

4. If $x > 0$, then simplify the expression $-3xy\sqrt[4]{32x^5y^6} + 2x^2\sqrt[4]{2^9xy^{10}}$.

5. Because of the curvature of the earth, the maximum distance D that you can see from the top of a tall building of height h is estimated by the formula

$$D = \sqrt{2rh + h^2}$$

where $r = 3960 \, mi$ is the radius of the earth and D and h are also measured in miles. How far can you see from the observation deck of the Toronto CN Tower, 1135 ft above the ground?


CN Tower
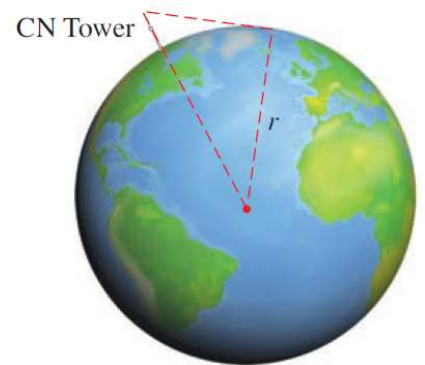
6. **OCEANOGRAPHY** The percent P of light that will pass to a depth d, in meters, at a certain place in the ocean is given by $P = 10^{2-(d/40)}$. Find, to the nearest percent, the amount of light that will pass to a depth of
a) 10 meters and   b) 25 meters below the surface of the ocean.

P.5 Recitation Exercises

1. Find the coefficient of x in the polynomial $(2x + 1)^2 - \left(3x + \frac{1}{2}\right)^3$.

**2.** If $X = (a - 2b)^3$ and $Y = (2a + b)^3$, then $X - Y =$

3. **Volume of a Box** An open box is constructed from a 6 $in.$ By 10 $in.$ sheet of cardboard by cutting a square piece from each corner and then folding up the sides, as shown in the figure.
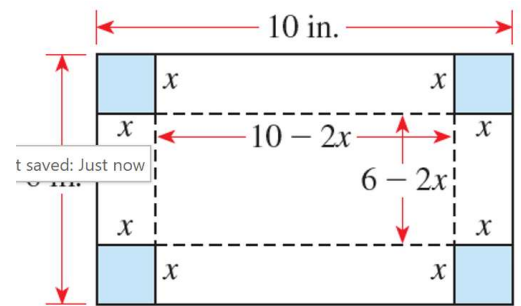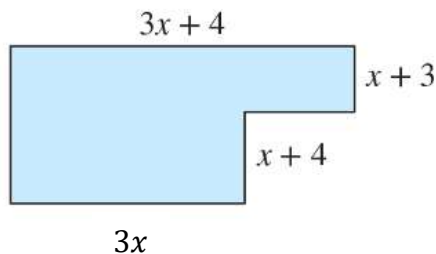
The volume of the box is

$$V = x(6 - 2x)(10 - 2x)$$

a) Explain how the expression for V is obtained.
b) Expand the expression for V. What is the degree of the resulting polynomial?
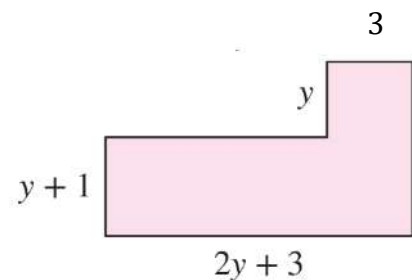c) Find the volume when $x = 1$ and when $x = 2$.
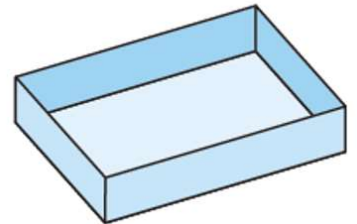
4. Write an expression that represents the perimeter and the area of each figure and simplify the expression

(A)

3x + 4

x + 3

x + 4

3x

(B)

3

y

y + 1

2y + 3

1. Factor completely the following expresssions.

a) $\left(1+\frac{1}{x}\right)^2 - \left(1-\frac{1}{x}\right)^2$

b) $y^3 - 1 - y^2 + y$

c) $2(a+b)^2 + 5(a+b) - 3$

d) $8r^3 - 64t^6$

e) $\frac{1}{2}x^{-1/2}(3x+4)^{1/2} + \frac{3}{2}x^{1/2}(3x+4)^{-1/2}$

2. One of the factors of $x^4 + x^2 - 2$ is

A) $x - 1$

B) $2x + 1$

C) $2x^2 + 1$

D) $x + \sqrt{2}$

E) $x - \sqrt{2}$

3. The possible value(s) of k that make(s) the trinomial $36x^2 + kxy + 49y^2$ a perfect square is (are)

A) 84   **B)** -84 **C)** ± 84 **D)** ± 42 **E)** -42

4. One of the factors of $4x^2 - 5y^2 - 4x + 10y - 8xy$ is

A) $2x + y - 2$ **B)** $4x - 4y - 2$ **C)** $2x + y$

D) $2x - y + 2$ **E)** $5x - 2y$

1. Simplify the following rational expressions.

a) $\dfrac{4}{2-x} + \dfrac{5}{x^2+2x+4} \div \dfrac{x^2-4x+4}{x^3-8}$

b) $\dfrac{x^{-1}+(x+2)^{-1}}{x^{-1}-(x+2)^{-1}}$

c) $1 + \dfrac{1}{1 + \dfrac{1}{1+x}}$

2. The expression $\dfrac{\frac{2x^2-3x-2}{x^2-1}}{\frac{2x^2+5x+2}{x^2+x-2}}$ simplifies to

A) $\dfrac{x+1}{x-2}$ **B)** $\dfrac{x-2}{x+1}$ **C)** $\dfrac{2x+1}{x+2}$ **D)** $\dfrac{x+2}{2x+1}$ **E)** $\dfrac{x+2}{x-1}$

3. If $\dfrac{\sqrt{1-x^2}+x^2(1-x^2)^{-\frac{1}{2}}}{(1-x^2)^{-\frac{3}{2}}} = ax^2 + b$, then $a + b =$

A) 2   **B)** -2 **C)** 0   **D)** 1   **E)** -1

4. Rationalize the denominator of $\dfrac{2x-2}{\sqrt{x}-1}$ and simplify.

5. Find the domain of $\dfrac{\sqrt{x}}{x^2-3x-4}$

6. Electrical Resistance If two electrical resistors with resistances R_1 and R_2 are connected in parallel (see the figure), then the total resistance R is given by

$$R = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2}}$$

a) Simplify the expression for R.

b) If $R_1 = 10$ ohms and $R_2 = 20$ ohms, what is the total resistance R ?

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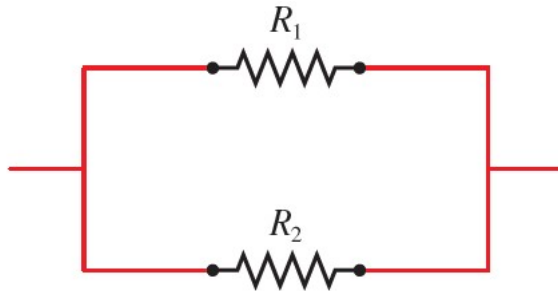
1. Solve the following equations for k.

$$-k = (5k + 3)(3x + 1)$$

b) $\dfrac{k+1}{b} = \dfrac{k-1}{b} + \dfrac{b+1}{k}$

2. Solve the following equations.

a) $\dfrac{1}{x} - \dfrac{2}{2x+1} = \dfrac{1}{2x^2+x}$

b) $6x^{\frac{2}{3}} - 216 = 0$

c) $(3x - 4)^2 - 7 = 0$

3. The difference between 5 times a number and 8 is equal 7 times the sum of the number and 3. Find the number.

4. If the length of each side of the original square is decreased by 4 inches, the perimeter of the new square is 10 inches more than half the perimeter of the original square. What are the dimensions of the original square?

5. In a jury pool there are 8 more men than women. If the ration of men to women is 8 to 7 , determine the number of men and women in the pool.

6. Sami participated in a 41-mi biathlon that included running and bicycling. He spent 1 hr 45 min on the bike and 45 min running. If his average speed on the bicycling was 12 mph faster then his average speed running, find his average speed running and his average speed riding.