11.1 Recitation Exercises

1. Determine whether the following systems is ***dependent, inconsistent*** or has ***a unique solution***

a) $\begin{cases} x - 2y + z = 1 \\ y + 2z = 5 \\ x + y + 3z = 8 \end{cases}$

b) $\begin{cases} x + y + z = 2 \\ y - 3z = 1 \\ 2x + y + 5z = 0 \end{cases}$

c) $\begin{cases} x + 2y + z = 1 \\ 5x + 2y + 3z = 4 \\ 3x - 2y + z = 2 \end{cases}$

2. Use the Gauss Jordan method to solve the following linear system

$$\begin{cases} x + y + 6z = 3 \\ x + y + 3z = 3 \\ x + 2y + 4z = 7 \end{cases}$$

3. If the echelon form of the linear system

$\begin{cases} x - 3y + z = 8 \\ 2x - 5y - 3z = 6 \\ x - 6y + 7z = -7 \end{cases}$ is $\left[\begin{array}{ccc|c} 1 & -3 & 1 & 8 \\ 0 & 1 & m & n \\ 0 & 0 & 1 & p \end{array}\right]$, then $(m,\ n,\ p) =$

A) $(-5\,,-10\,,5)$

B) $(3,-6,-3)$

C) $(-5,10,-3)$

D) $(-2\,,7,-1)$

E) $(-3,6,-2)$

11.2 Recitation Exercises

1. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$, and $B = \begin{bmatrix} 2 & 3 & 5 \\ 0 & -4 & 2 \\ 0 & 0 & -3 \end{bmatrix}$, then find

a) $A + B$ **b)** BA **c)** B^2

2. If $A, B \text{ and } C$ are *square* matrices and I_n is the identity matrix, which one of the following statement is True?

A) $(A + B)(A^2 - AB + B^2) = A^3 + B^3$

B) $(A + I_n)(A - I_n) = A^2 - I_n$

C) $(A - B)^2 = A^2 - 2AB + B^2$

D) $AB = 0 \text{ implies } A = 0 \text{ or } B = 0$

E) $I_n A = I_n$

3. If $A = \begin{bmatrix} 1 & 3 & 4 \\ -2 & 2 & 5 \\ 1 & 3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 6 & 0 & 2 \\ 0 & 1 & 3 \\ -1 & 2 & 5 \end{bmatrix}$, and $C = AB$, then $c_{32} + c_{13} =$

A) 52 B) 11 **C)** 38 **D)** -15 **E)** 9

4. If $A = \begin{bmatrix} 1 & 2 & 4 \\ -1 & 0 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 2 & 1 \\ 3 & -1 & 2 \end{bmatrix}$, then find the matrix X that satisfies $4X + B = 2X + 3A$.

11.3 Recitation Exercises

1. Find the inverse of the matrix if it exists

a) $A = \begin{bmatrix} 3 & 2 \\ 6 & 4 \end{bmatrix}$.

b) $B = \begin{bmatrix} 2 & 4 & 1 \\ -1 & 1 & -1 \\ 1 & 4 & 0 \end{bmatrix}$.

2. Use the inverse of the coefficient matrix to solve the following system

$$\begin{cases} 2x + y = -7 \\ 3x + 2y = 19 \end{cases}$$

3. Use the inverse of the coefficient matrix (if possible) to solve the following systems

a) $\begin{cases} x + 2y + 3z = 1 \\ 4x + 5y - z = 2 \\ x - y - 10z = -3 \end{cases}$

b) $\begin{cases} x + 2y + 3z = 1 \\ 4x + 5y - z = 3 \\ x - y - 10z = 0 \end{cases}$

4. Given that $M^{-1} = \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$ and $N^{-1} = \begin{bmatrix} -1 & -1 \\ 4 & 3 \end{bmatrix}$, find the sum of the elements in the second column of $(MN)^{-1}$.

11.4 Recitation Exercises

1. If $A = \begin{bmatrix} 2 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 6 & 2 & 2 \\ 1 & 1 & -2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 & 2 & 1 \\ 3 & 0 & 1 & -1 \\ -1 & 2 & -2 & 1 \\ -3 & 2 & 3 & 2 \end{bmatrix}$, then

a) Find the sum of the cofactors of A_{23} and B_{44}.

b) Find |A|.

2. Evaluate the following determinants

a) $\begin{vmatrix} 3 & 5 \\ 2 & 4 \end{vmatrix}$ **b)** $\begin{vmatrix} 2 & 0 & 0 \\ 4 & 1 & 0 \\ 7 & 3 & -2 \end{vmatrix}$ **c)** $\begin{vmatrix} 3 & 0 & 0 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{vmatrix}$

c) $\begin{vmatrix} 4 & 0 & 2 & 1 \\ 5 & 0 & 4 & 2 \\ 2 & 0 & 3 & 4 \\ 1 & 0 & 2 & 3 \end{vmatrix}$ **e)** $\begin{vmatrix} 5 & -13 & -3 \\ -2 & 5 & 1 \\ -2 & 6 & 2 \end{vmatrix}$

3. Let A and B be 4×4 invertible matrices. Determine wither each of the following statements is true or false:

a) $|A^2| = |A|^2$ **b)** $|2B| = 8|B|$ **c)** $|A.B| = |A|.|B|$

d) $|A + B| = |A| + |B|$ **e)** $|I_n| = 1$ **f)** $|A^{-1}| = |A|$.

4. If $\det(M - xI) = 0$, $M = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then $x =$

A) 1 **B)** 2 **C)** 3 **D)** -1 **E)** -1/2.

5. If A and B are 3×3 matrices such that $|A| = 5$ and $|B| = -2$, then

$|3(A\,B^2)^{-1}| =$

A) $\frac{27}{20}$ **B)** $\frac{15}{10}$ **C)** $\frac{-30}{4}$ **D)** $\frac{10}{27}$ **E)** 540