CHAPTER 9.2

The Dot Product

Multiple-Choice Old Exam Questions

27 questions • 5 topics • options A–E • two questions per page

Topic 1 | Finding the Angle Between Two Vectors

Finding the angle between two vectors, given in component form or in terms of i and j, or from magnitude and direction • Questions 1–6

Q **1**

The smallest positive angle between the vectors $\mathbf{u} = \langle 2, -2\sqrt{3} \rangle$ and $\mathbf{v} = -2\sqrt{3}\mathbf{i} + 2\mathbf{j}$ is

A) $120°$

B) $135°$

C) $30°$

D) $60°$

E) $150°$

Q **2**

For the vectors $\mathbf{u} = \langle 0, 5 \rangle$ and $\mathbf{v} = \langle -2, 2 \rangle$, the smallest positive angle between the vectors $\mathbf{u} + \mathbf{i}$ and $\mathbf{v} + \mathbf{j}$ is

A) $\cos^{-1} \frac{1}{2}$

B) $\cos^{-1} \left(-\frac{2}{\sqrt{13}} \right)$

C) $120°$

D) $45°$

E) $135°$

Q
3

The smallest positive angle between the vectors $\mathbf{u} = \cos\left(\frac{\pi}{2}\right)\mathbf{i} + \sin\left(\frac{\pi}{2}\right)\mathbf{j}$ and $\mathbf{w} = \cos\left(\frac{3\pi}{4}\right)\mathbf{i} + \sin\left(\frac{3\pi}{4}\right)\mathbf{j}$ is equal to

A) $75°$

B) $15°$

C) $105°$

D) $45°$

E) $30°$

Q
4

The angle between the vectors $\mathbf{u} = \langle 2, 1\rangle$ and $\mathbf{v} = -3\mathbf{i} + \mathbf{j}$ is equal to

A) $135°$

B) $210°$

C) $45°$

D) $120°$

E) $150°$

Q
5

If α is the smallest positive angle between the vectors $\mathbf{u} = -\mathbf{i} + 5\mathbf{j}$ and $\mathbf{v} = 4\mathbf{i} + 6\mathbf{j}$, then $\alpha =$

A) $45°$

B) $60°$

C) $135°$

D) $120°$

E) $30°$

Q
6

Let $\mathbf{u}$ and $\mathbf{w}$ be two vectors such that $\mathbf{u} = 2\mathbf{i} + 2\sqrt{3}\mathbf{j}$ and $\mathbf{w}$ has magnitude 3 and direction angle $120°$. Then the smallest angle between $\mathbf{u}$ and $\mathbf{w}$ is

A) $60°$

B) $30°$

C) $45°$

D) $120°$

E) $150°$

Topic 2 | Trigonometric Ratios of the Angle Between Vectors

Finding the sine, cosine, tangent, cotangent or secant of the angle between two vectors • Questions 7–14

Q **7**

If θ is the smallest angle between the vectors $\mathbf{u} = \langle 2,\ 1\rangle$ and $\mathbf{v} = \langle -3,\ 1\rangle$, then $\sin\theta =$

A) $\frac{\sqrt{2}}{2}$

B) $-\frac{\sqrt{2}}{2}$

C) $\frac{\sqrt{3}}{2}$

D) $\frac{1}{2}$

E) $-\frac{1}{2}$

Q **8**

The cosine of the smallest positive angle between the vectors $\mathbf{u} = \langle -1,\ 1\rangle$ and $\mathbf{v} = \langle 1,\ 7\rangle$ is equal to

A) $\frac{3}{5}$

B) $\frac{7}{10}$

C) $-\frac{1}{5}$

D) $\frac{4}{5}$

E) $\frac{6}{5\sqrt{2}}$

Q
9

If α is the smallest positive angle between the two vectors $\mathbf{u} = 4\mathbf{i} - 3\mathbf{j}$ and $\mathbf{v} = \langle 4, 1 \rangle$, then $\cos\alpha =$

A) $\frac{13\sqrt{17}}{85}$

B) $\frac{\sqrt{17}}{13}$

C) $\frac{17}{85}$

D) $\frac{13}{85}$

E) $\frac{12}{17}$

Q
10

If α is the angle between the vectors $\mathbf{u} = \mathbf{i} + 3\mathbf{j}$ and $\mathbf{v} = -\mathbf{i} + 3\mathbf{j}$, then $\tan\alpha =$

A) $\frac{3}{4}$

B) $-\frac{3}{4}$

C) $-\frac{3}{5}$

D) $-\frac{4}{5}$

E) $\frac{3}{5}$

Q
11

If θ is the smallest positive angle between the two vectors $\mathbf{u} = \langle 3, 4 \rangle$ and $\mathbf{v} = 2\mathbf{i} + \mathbf{j}$, then $\sec\theta =$

A) $\frac{\sqrt{5}}{2}$

B) $\frac{2\sqrt{5}}{5}$

C) $\frac{2}{5}$

D) $\frac{5}{2}$

E) $\frac{3}{4}$

Q
12

If α is the angle between the vectors $3\mathbf{i} + 4\mathbf{j}$ and $\mathbf{j}$, where $0° \leq \alpha \leq 180°$, then $\sin\alpha =$

A) $\frac{3}{5}$

B) $\frac{4}{5}$

C) $-\frac{3}{5}$

D) $-\frac{4}{5}$

E) $\frac{3}{4}$

Q
13

If α is the smallest angle between the vectors $\mathbf{u} = \langle 3, -2 \rangle$ and $\mathbf{v} = \langle 2, -2 \rangle$, then $\cos^2 \alpha =$

A) $\frac{25}{26}$

B) $\frac{1}{26}$

C) $\frac{1}{13}$

D) $\frac{2}{13}$

E) $\frac{7}{13}$

Q
14

If α is the smallest positive angle between the vectors $\mathbf{u} = \langle 3, -4 \rangle$ and $\mathbf{v} = \langle -2, 1 \rangle$, then $\cot \alpha =$

A) -2

B) $-\frac{2}{5}$

C) -3

D) $\frac{2}{5}$

E) $\frac{1}{2}$

Topic 3 | Dot Product, Magnitude and Operations

Using the dot product to find u · v, a magnitude or a dot product of combinations of vectors • Questions 15–18

Q **15**

Let $\mathbf{u}$ and $\mathbf{v}$ be two vectors. If $|\mathbf{u}| = 4$, $|\mathbf{v}| = 4$ and $|\mathbf{u} + \mathbf{v}| = 5\sqrt{2}$, then $\mathbf{u} \cdot \mathbf{v} =$

A) 7

B) 8

C) 16

D) 9

E) 6

Q **16**

If $\mathbf{u}$ and $\mathbf{v}$ are unit vectors and the angle between $\mathbf{u}$ and $\mathbf{v}$ is $120°$, then $|\mathbf{u} - \mathbf{v}|$ is equal to

A) $\sqrt{3}$

B) 5

C) $\sqrt{2}$

D) 0

E) $\frac{1}{2}$

Q
17

Let $a = \cos^{-1}\left(-\frac{2}{\sqrt{5}}\right)$ be the smallest positive angle between the vectors $\mathbf{u}$ and $\mathbf{v}$. If $|\mathbf{u}| = 5$ and $|\mathbf{v}| = \sqrt{5}$ are the magnitudes of $\mathbf{u}$ and $\mathbf{v}$, then the dot product $\mathbf{u} \cdot \mathbf{v} =$

A) -10

B) $-\frac{25}{2}$

C) $-\frac{1}{2}$

D) -5

E) $-\sqrt{5}$

Q
18

If the vectors $\mathbf{u} = \langle \sin 20°, \cos 20° \rangle$ and $\mathbf{v} = \langle \cos 80°, -\sin 80° \rangle$, then $\mathbf{u} \cdot \mathbf{v} =$

A) $-\frac{\sqrt{3}}{2}$

B) $-\frac{1}{2}$

C) $\frac{1}{2}$

D) $\cos 100°$

E) $-\sin 100°$

Topic 4 | Orthogonal (Perpendicular) Vectors

Using u · v = 0 to find an unknown constant in perpendicular vectors • Questions 19–24

Q **19**

Let $\mathbf{u} = \langle 2, -1 \rangle$, $\mathbf{v} = \langle 1, -2 \rangle$ and $\mathbf{w} = 12\mathbf{i} + a\mathbf{j}$. If $\mathbf{w}$ is orthogonal to the vector $-2\mathbf{u} + 3\mathbf{v}$, then $a =$

A) -3

B) 2

C) -6

D) 1

E) 4

Q **20**

Let $\mathbf{u}$ and $\mathbf{v}$ be two vectors such that $\mathbf{u} = k\mathbf{i} - \mathbf{j}$ and $\mathbf{v}$ is a vector of magnitude $\frac{\sqrt{2}}{2}$ and direction angle $\frac{3\pi}{4}$. If $\mathbf{u}$ and $\mathbf{v}$ are perpendicular, then $k =$

A) -1

B) 1

C) $-\frac{1}{2}$

D) $\frac{1}{2}$

E) 2

Q
21

If $\mathbf{u} = \cos\frac{3\pi}{4}\mathbf{i} + \sin\frac{3\pi}{4}\mathbf{j}$ and $\mathbf{v} = \langle 4k + 1,\ k - 3\rangle$ are perpendicular, then $k =$

A) $-\frac{4}{3}$

B) $\frac{5}{4}$

C) $\frac{4}{5}$

D) $\frac{2}{5}$

E) $-\frac{2}{3}$

Q
22

If the vectors $\mathbf{u} = \frac{5r}{7}\mathbf{i} + \frac{1}{3}\mathbf{j}$ and $\mathbf{v} = \left\langle \frac{r}{5},\ -\frac{2}{7}\right\rangle$ are orthogonal, then a possible value of r is

A) $\frac{\sqrt{6}}{3}$

B) $\frac{\sqrt{3}}{3}$

C) $\frac{\sqrt{2}}{2}$

D) $\frac{\sqrt{6}}{2}$

E) $\frac{\sqrt{3}}{2}$

Q
23

Let $\mathbf{u}$ and $\mathbf{v}$ be two vectors such that $\mathbf{u} = -\sqrt{3}\mathbf{i} - k\mathbf{j}$ and $\mathbf{v}$ is a vector with magnitude 2 and direction angle $150°$. If $\mathbf{u}$ and $\mathbf{v}$ are perpendicular vectors, then the value of k is

A) 3

B) 2

C) -1

D) 4

E) 0

Q
24

If the vectors $\mathbf{u} = (k-1)\mathbf{i} + \mathbf{j}$ and $\mathbf{v} = 3\mathbf{i} + (k+1)\mathbf{j}$ are perpendicular, then k is equal to

A) $\frac{1}{2}$

B) $\frac{5}{8}$

C) 2

D) 4

E) $\frac{1}{4}$

Topic 5 | True and False Statements

Deciding which statement about vectors and the dot product is true or false • Questions 25–27

Q **25**

Which one of the following statements is TRUE?

A) If $\mathbf{v} = \left\langle -\frac{4}{5}, -\frac{3}{5} \right\rangle$, then $\mathbf{v}$ is a unit vector.

B) If $\mathbf{u} = \langle 3, 2 \rangle$ and $\mathbf{v} = \langle -1, 1 \rangle$, then $\mathbf{u}$ and $\mathbf{v}$ are perpendicular.

C) If $\mathbf{u} = \langle 3, 2 \rangle$, then it can be written as $\mathbf{u} = 2\mathbf{i} + 3\mathbf{j}$.

D) If $\mathbf{u} = \langle 3, 2 \rangle$ and $\mathbf{v} = \langle 1, 3 \rangle$, then $\mathbf{u} \cdot \mathbf{v} = 3$.

E) If α is the angle between the vectors $\mathbf{u}$ and $\mathbf{v}$, then $\tan \alpha = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}||\mathbf{v}|}$.

Q **26**

For the vectors $\mathbf{s}$, $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ and the real number k, which one of the following statements is FALSE?

A) $\mathbf{s} = \langle 1, 1 \rangle$ is a unit vector

B) $\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$

C) $\mathbf{u} \cdot (\mathbf{v} + \mathbf{w}) = \mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w}$

D) $(k\mathbf{u}) \cdot \mathbf{v} = \mathbf{u} \cdot (k\mathbf{v})$

E) $\mathbf{u} \cdot \mathbf{u} = |\mathbf{u}|^2$

Q
27

Which one of the following statements is TRUE?

A) The vector $\langle \sin 25°, \sin 65° \rangle$ is a unit vector.

B) The vectors $\langle -1, 1 \rangle$ and $\langle 2, -2 \rangle$ are perpendicular.

C) The vectors $\langle 1, -1 \rangle$ and $\langle 2, -2 \rangle$ have the same magnitude.

D) The vectors $\langle -4, -4 \rangle$ and $\langle 4, 4 \rangle$ have the same direction.

E) The dot product of two vectors is a vector.

Answer Key

"Original" shows the question number in the source worksheet.

Q	Original	Ans.	Value
1	2	**E**	$150°$
2	5	**D**	$45°$
3	6	**D**	$45°$
4	14	**A**	$135°$
5	17	**A**	$45°$
6	18	**A**	$60°$
7	1	**A**	$\frac{\sqrt{2}}{2}$
8	7	**A**	$\frac{3}{5}$
9	8	**A**	$\frac{13\sqrt{17}}{85}$
10	9	**A**	$\frac{3}{4}$
11	10	**A**	$\frac{\sqrt{5}}{2}$
12	11	**A**	$\frac{3}{5}$
13	12	**A**	$\frac{25}{26}$
14	13	**A**	-2
15	3	**D**	9
16	4	**A**	$\sqrt{3}$
17	19	**A**	-10
18	21	**A**	$-\frac{\sqrt{3}}{2}$
19	22	**A**	-3
20	23	**A**	-1
21	24	**A**	$-\frac{4}{3}$
22	25	**A**	$\frac{\sqrt{6}}{3}$
23	26	**A**	3
24	27	**A**	$\frac{1}{2}$
25	15	**A**	If $\mathbf{v} = \left\langle -\frac{4}{5}, -\frac{3}{5} \right\rangle$, then $\mathbf{v}$ is a unit vector.
26	16	**A**	$\mathbf{s} = \langle 1, 1 \rangle$ is a unit vector
27	20	**A**	The vector $\langle \sin 25°, \sin 65° \rangle$ is a unit vector.