CHAPTER 7.4 - 7.5

Basic and More Trigonometric Equations

Multiple-Choice Old Exam Questions

52 questions • 5 topics • options A–E • two questions per page

Topic 1 | Quadratic-Type Trigonometric Equations

Solving equations that become quadratic in sine, cosine or tangent, and finding a sum, product, count or value • Questions 1–12

Q **1**

The product of all solutions of the equation $\cos x - 2\sin^2 x + 1 = 0$ in the interval $[0,\ 2\pi)$ is

A) $\frac{\pi^2}{3}$

B) $\frac{5\pi^3}{9}$

C) $\frac{11\pi^3}{36}$

D) $\frac{\pi^2}{6}$

E) $\frac{5\pi^3}{36}$

Q **2**

The sum of all the solutions of the equation $2\cos^2 x + |\cos x| - 1 = 0$, $0 < x \leq 2\pi$, is

A) 4π

B) 3π

C) $\frac{5\pi}{4}$

D) π

E) 5π

Q
3

If θ is the solution of $4\sin^2\theta + 2\cos^2\theta = 3, \frac{3\pi}{2} \leq \theta < 2\pi$, then $\sec\theta =$

A) $\sqrt{2}$

B) 2

C) $-\sqrt{2}$

D) $\frac{2\sqrt{3}}{3}$

E) $-\frac{2\sqrt{3}}{3}$

Q
4

The number of solutions of $(\cos x)(\cos x + 1) = 2, 0 \leq x < 2\pi$, is equal to

A) 4

B) 1

C) 2

D) 3

E) 5

Q
5

The sum of all the solutions of the equation $2\sin\theta\,(\sin\theta - 1) = 3(\sin\theta + 1)$, $0 \leq \theta < 360°$, is

A) $540°$

B) $420°$

C) $360°$

D) $240°$

E) $620°$

Q
6

If $0 \leq x < 2\pi$, then the sum of all solutions of the equation $\sin^2 x - \sin x - 2 = 0$ is equal to

A) $\frac{3\pi}{2}$

B) π

C) $\frac{5\pi}{3}$

D) 2π

E) $\frac{5\pi}{2}$

Q
7

The number of solutions of the equation $4\sin^2 x + 2\sqrt{3}\sin x - \sqrt{3} = 2\sin x$, $0 \leq x < 2\pi$, is

A) 4

B) 5

C) 3

D) 2

E) 1

Q
8

The number of solutions of $\tan^2\theta = \frac{3}{2}\sec\theta$ on the interval $[0, 2\pi)$ is equal to

A) 2

B) 3

C) 1

D) 4

E) 5

Q
9

The number of solutions of the equation $3 + \cos 2\,\theta = 5\cos\theta$, $0 \leq \theta \leq 4\pi$, is equal to

A) 8

B) 10

C) 4

D) 6

E) 2

Q
10

The sum of all solutions of the equation $2\sin^2 x - \cos x = 1$ in the interval $[0,\ 3\pi)$ is

A) 2π

B) $\frac{16\pi}{3}$

C) $\frac{11\pi}{12}$

D) 3π

E) $\frac{11\pi}{6}$

Q
11

The number of solutions of the equation $2\cos^2 3\theta - 2\cos 3\theta - \sqrt{3}\cos 3\theta + \sqrt{3} = 0, 0 \leq \theta < 2\pi$, is

A) 9

B) 4

C) 5

D) 6

E) 3

Q
12

The sum of all solutions of the equation $4\cos^2 \theta - 4\cos\theta = -1, 0 \leq \theta < 2\pi$, is

A) 2π

B) π

C) 3π

D) 4π

E) 0

Topic 2 | Sums of Solutions on an Interval

Finding the sum of all solutions of an equation in a given interval • Questions 13–28

Q **13**

The sum of all the solutions of the equation $\cot\theta + \tan\theta = 2, 0 \leq \theta < 2\pi$, is equal to

A) $\frac{3\pi}{2}$

B) $\frac{3\pi}{4}$

C) 2π

D) $\frac{\pi}{2}$

E) π

Q **14**

The sum of the solutions of the equation $\cos 2x - \cos x = 0$ on the interval $[0, 2\pi)$ is

A) 2π

B) π

C) 4π

D) 3π

E) 5π

Q
15

The sum of all solutions of the equation $\sin 2x + 2\sqrt{3}\cos x - \sin x = \sqrt{3}$ over the interval $[0, 2\pi)$ is

A) 2π

B) 5π

C) 3π

D) 4π

E) 6π

Q
16

The sum of all the solutions of $\sin\left(\theta + \frac{\pi}{4}\right) + \sin\left(\theta - \frac{\pi}{4}\right) = -1, 0 \leq \theta < 2\pi$, is equal to

A) 3π

B) 5π

C) π

D) 4π

E) 1.5π

Q
17

The sum of all the solutions of the equation $2\sin x\cos x - 2\sin x + \cos x = 1$ in the interval $[0, 2\pi)$ is

A) 3π

B) $\frac{5\pi}{3}$

C) 2π

D) $\frac{4\pi}{3}$

E) $\frac{7\pi}{3}$

Q
18

The sum of all the solutions of the equation $2\sin x\cos x + 3\cos x = 0$ in the interval $[0, 2\pi)$ is equal to

A) 2π

B) $\frac{5\pi}{2}$

C) 3π

D) π

E) $\frac{3\pi}{2}$

Q
19

The sum of the solution(s) of the equation $\sqrt{2}\sec\frac{x}{2}+2=0$ in the interval $[0,\ 2\pi)$ is

A) $\frac{3\pi}{2}$

B) $\frac{3\pi}{4}$

C) 4π

D) 2π

E) π

Q
20

The sum of the solutions of the equation $\csc^2 x-2\cot x=0$ in the interval $[0,\ 2\pi)$ is

A) $\frac{3\pi}{2}$

B) $\frac{5\pi}{2}$

C) $\frac{5\pi}{4}$

D) π

E) $\frac{9\pi}{4}$

Q
21

The sum of the solutions of the equation $2\sin x\cos x + 2\sin x - \cos x - 1 = 0$ over the interval $[0, 2\pi)$ is

A) 2π

B) π

C) $\frac{3\pi}{2}$

D) 3π

E) $\frac{7\pi}{6}$

Q
22

The sum of solutions of the equation $2\tan x = \sec^2 x$ in the interval $[0, 2\pi)$ is equal to

A) $\frac{3\pi}{2}$

B) $\frac{\pi}{4}$

C) 5π

D) π

E) $\frac{3\pi}{4}$

Q
23

The sum of all the solutions of the equation $2\cos x = \sin^2 x - 2$ in $[0, 2\pi)$ is equal to

A) π

B) 2π

C) 3π

D) $\frac{\pi}{2}$

E) $\frac{3\pi}{2}$

Q
24

The sum of all solutions of the equation $4\sin 2x + \sin 4x = 0, 0 \leq x < 2\pi$, is

A) 3π

B) $\frac{7\pi}{3}$

C) $\frac{13\pi}{6}$

D) $\frac{5\pi}{2}$

E) 5π

Q
25

The sum of all the solutions of the equation $\tan x \cos x - \tan x - \cos x + 1 = 0$ in $[0, 2\pi)$ is equal to

A) $\frac{3\pi}{2}$

B) $\frac{\pi}{4}$

C) $\frac{13\pi}{4}$

D) $\frac{13\pi}{12}$

E) $\frac{5\pi}{2}$

Q
26

The sum of all the solutions of $\sin 2x \cos x + \cos 2x \sin x = 0, 0 \leq x < \pi$, is equal to

A) 5π

B) π

C) 4π

D) 2π

E) 3π

Q
27

The sum of all solutions of the equation $\tan\frac{x}{2} = \sin x$, $0 \leq x < 2\pi$, is equal to

A) 2π

B) $\frac{\pi}{2}$

C) $\frac{3\pi}{2}$

D) π

E) 3π

Q
28

The sum of all solutions of the equation $\sin 3x \cos x - \cos 3x \sin x - \frac{1}{2} = 0$, where $0 \leq x < 2\pi$, is

A) $\frac{4\pi}{3}$

B) 0

C) $\frac{5\pi}{12}$

D) π

E) 3π

Topic 3 | Number of Solutions

Counting the solutions of an equation in a given interval • Questions 29–40

Q
29

If $0 \leq x < 2\pi$, then the number of all solutions of the equation $2\sin\left(2x + \frac{\pi}{6}\right) - 1 = 0$ is

A) 4

B) 3

C) 5

D) 6

E) 8

Q
30

The number of solutions of the equation $\tan x + \sqrt{3} = \sec x$ in the interval $[0, 360°)$ is

A) One

B) Two

C) Three

D) Four

E) Five

Q
31

The number of all solutions of the equation $\sin x - \cos x = 2, 0 \leq x < 2\pi$, is

A) 0

B) 2

C) 3

D) 4

E) 1

Q
32

The number of solutions of the equation $4 \sin x \cos x = \sqrt{2}, 0 \leq x < 2\pi$, is

A) 4

B) 1

C) 2

D) 3

E) 5

Q
33

If $0° \leq x < 360°$, then the number of solutions of the equation $4 \tan x \sin^2 x + \tan x - 4\sqrt{3} \sin^2 x - \sqrt{3} = 0$ is equal to

A) 2

B) 3

C) 4

D) 5

E) 6

Q
34

The equation $\cos x \cos 3x + \sin x \sin 3x = \frac{1}{2}$, $-\pi \leq x < 0$, has

A) 2 solutions

B) 1 solution

C) 3 solutions

D) 4 solutions

E) 5 solutions

Q
35

If $0° \leq x < 360°$, then the number of solutions of the equation $4 \sin x \cos x + 5 \cos x = 0$ is equal to

A) 2

B) 1

C) 4

D) 0

E) 3

Q
36

The number of solutions of the equation $\tan \frac{x}{2} = \sin x$, $0 \leq x < 2\pi$, is

A) 3

B) 2

C) 1

D) 4

E) 5

Q
37

The number of solutions of the equation $\sqrt{3}\cot^2 x + \cot x - 1 - \sqrt{3}\cot x = 0$ over the interval $\left[0, \frac{3\pi}{2}\right]$ is

A) 3

B) 1

C) 2

D) 4

E) 0

Q
38

The equation $\sin x + \cos x = 2, 0 \leq x < 2\pi$, has

A) no solution

B) 3 solutions

C) 1 solution

D) 4 solutions

E) 2 solutions

Q
39

The number of solutions of the equation $4\cos\theta\sin\theta = \sqrt{3}$ over the interval $[0°, 360°)$ is

A) 4

B) 5

C) 3

D) 2

E) 1

Q
40

The number of all solutions of the equation $\sin\theta = \sec\theta$ for $0 \leq \theta < 2\pi$ is

A) 0

B) 1

C) 2

D) 3

E) 4

Topic 4 | Equations with Double and Half Angles

Using double-angle and half-angle identities to solve equations • Questions 41–48

Q **41**

If θ is the solution of the equation $2\tan\theta + \sec^2\theta = 4, \frac{3\pi}{2} < \theta < 2\pi$, then $\cos\theta =$

A) $\frac{\sqrt{10}}{10}$

B) $\frac{\sqrt{5}}{5}$

C) $\frac{\sqrt{3}}{3}$

D) $\frac{\sqrt{6}}{6}$

E) $\frac{\sqrt{2}}{2}$

Q **42**

The sum of solutions of the equation $\cos x \cos 2x - \sin x \sin 2x = 0$, $0 \leq x < \pi$, is

A) $\frac{7\pi}{6}$

B) $\frac{5\pi}{2}$

C) $\frac{3\pi}{2}$

D) $\frac{5\pi}{6}$

E) 3π

Q
43

The sum of all solutions of the equation $-2\cos 2x \sin 3x + 2\cos 3x \sin 2x = \sqrt{3}$ in the interval $[-\pi, \pi]$ is

A) $-\frac{4\pi}{3}$

B) $-\frac{2\pi}{3}$

C) $\frac{2\pi}{3}$

D) $-\pi$

E) $\frac{\pi}{3}$

Q
44

The sum of solutions of the equation $2\sin^2\frac{x}{2} = \cos x$ over the interval $[0, 2\pi)$ is equal to

A) 2π

B) π

C) 3π

D) 4π

E) $-\pi$

Q
45

If $0° \leq x < 360°$, then the sum of all solutions of the equation $4\cos 2x = 8\sin x \cos x$ is equal to

A) $630°$

B) $135°$

C) $505°$

D) $545°$

E) $475°$

Q
46

Let n be any integer. Then all solutions of the equation $\frac{4\tan\theta}{1-\tan^2\theta} - 2 = 0$ are

A) $(1+4n)\frac{\pi}{8}$

B) $(1+3n)\frac{\pi}{4}$

C) $(1+4n)\frac{3\pi}{4}$

D) $(1+3n)\frac{\pi}{8}$

E) $(1+3n)\frac{\pi}{6}$

Q
47

The solution set of $\sqrt{3}\sin x + \cos x - 1 = 0$ is equal to

A) $\left\{2\pi\, k,\ \frac{2\pi}{3} + 2k\pi\right\}$, where k is an integer

B) $\left\{\frac{\pi}{4} + 2\pi\, k,\ \frac{5\pi}{4} + 2k\pi\right\}$, where k is an integer

C) $\left\{2\pi\, k,\ \frac{2\pi}{3} + 2k\pi,\ \frac{4\pi}{3} + 2k\pi\right\}$, where k is an integer

D) $\{2\pi\, k\}$, where k is an integer

E) $\left\{2\pi\, k,\ \frac{5\pi}{3} + 2k\pi\right\}$, where k is an integer

Q
48

The solution set of the equation $\cot^2 x - \csc x - 1 = 0,\ 0 \leq\ x < 2\pi$, is:

A) $\left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}\right\}$

B) $\left\{\frac{\pi}{6}, \frac{7\pi}{6}, \frac{\pi}{2}\right\}$

C) $\left\{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{3\pi}{2}\right\}$

D) $\left\{\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$

E) $\left\{\frac{\pi}{6}, \frac{7\pi}{6}, \pi\right\}$

Topic 5 | Equations with Inverse Trigonometric Functions

Solving equations that involve inverse trigonometric functions • Questions 49–52

Q **49**

The sum of all solutions of the equation $\sin^{-1} x + \tan^{-1} x = 0$ is

A) zero

B) a negative irrational number

C) a positive irrational number

D) a negative integer

E) a positive integer

Q **50**

If $\cos^{-1} x = 2 \sin^{-1} \frac{1}{2}$, then $x =$

A) $\frac{1}{2}$

B) $\frac{\sqrt{3}}{2}$

C) $-\frac{1}{2}$

D) $-\frac{\sqrt{3}}{2}$

E) 1

Q
51

The solution of the equation $\cos^{-1} x + \tan^{-1} \frac{5}{12} = \frac{\pi}{2}$ is

A) $\frac{5}{13}$

B) $-\frac{12}{13}$

C) $\frac{13}{12}$

D) $-\frac{13}{5}$

E) $\frac{7}{12}$

Q
52

The solution set of the equation $2\cos^{-1}\left(\frac{x-\pi}{3}\right) = 2\pi$ is

A) $\{\pi - 3\}$

B) $\{\pi + 2\}$

C) $\{4 - \pi\}$

D) $\{2\pi - 3\}$

E) $\{\pi\}$

Answer Key

"Original" shows the question number in the source worksheet.

Q	Original	Ans.	Value
1	1	**B**	$\frac{5\pi^3}{9}$
2	2	**A**	4π
3	4	**A**	$\sqrt{2}$
4	5	**B**	1
5	9	**A**	$540°$
6	21	**A**	$\frac{3\pi}{2}$
7	22	**A**	4
8	23	**A**	2
9	37	**C**	4
10	38	**B**	$\frac{16\pi}{3}$
11	41	**A**	9
12	54	**A**	2π
13	3	**A**	$\frac{3\pi}{2}$
14	10	**A**	2π
15	11	**A**	2π
16	14	**A**	3π
17	17	**A**	3π
18	19	**A**	2π
19	26	**A**	$\frac{3\pi}{2}$
20	27	**A**	$\frac{3\pi}{2}$
21	30	**A**	2π
22	31	**A**	$\frac{3\pi}{2}$
23	33	**A**	π
24	46	**A**	3π
25	47	**A**	$\frac{3\pi}{2}$
26	6	**B**	π
27	12	**A**	2π
28	20	**E**	3π
29	15	**A**	4
30	16	**A**	One
31	18	**A**	0
32	24	**A**	4
33	25	**A**	2
34	28	**A**	2 solutions

Q	Original	Ans.	Value
35	29	**A**	2
36	32	**A**	3
37	34	**A**	3
38	40	**A**	no solution
39	48	**A**	4
40	53	**A**	0
41	13	**A**	$\frac{\sqrt{10}}{10}$
42	36	**C**	$\frac{3\pi}{2}$
43	39	**D**	$-\pi$
44	42	**A**	2π
45	43	**A**	$630°$
46	35	**A**	$(1+4n)\frac{\pi}{8}$
47	44	**A**	$\left\{2\pi\ k,\ \frac{2\pi}{3}+2k\pi\right\}$, where k is an integer
48	45	**A**	$\left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}\right\}$
49	7	**A**	zero
50	8	**A**	$\frac{1}{2}$
51	49	**A**	$\frac{5}{13}$
52	51	**A**	$\{\pi-3\}$