CHAPTER 7.3

Double-Angle and Half-Angle Formulas

Multiple-Choice Old Exam Questions

53 questions • 5 topics • options A–E • two questions per page

Topic 1 | Half-Angle Values from a Given Ratio

Using a given ratio and the quadrant to find sine, cosine, tangent or secant of a half angle • Questions 1–8

Q 1

If $\tan\alpha = -\frac{4}{3}, \frac{3\pi}{2} < \alpha < 2\pi$, then $\sec\frac{\alpha}{2} =$

A) $-\frac{\sqrt{5}}{2}$

B) $\frac{2}{\sqrt{3}}$

C) $\frac{2}{\sqrt{5}}$

D) $-\frac{\sqrt{3}}{2}$

E) $-\sqrt{5}$

Q 2

If $\sin\theta = \frac{2\sqrt{2}}{3}, 0 < \theta < \frac{\pi}{2}$, then $\sin^2\frac{\theta}{2} =$

A) $\frac{1}{3}$

B) $\frac{1}{2}$

C) $\frac{2}{3}$

D) 3

E) 2

Q
3

If $\csc\theta = -\frac{5}{4}$, where $\frac{3\pi}{2} < \theta < 2\pi$, then $\sec\frac{\theta}{2} =$

A) $-\frac{\sqrt{5}}{2}$

B) $-\frac{\sqrt{5}}{5}$

C) $\frac{\sqrt{5}}{5}$

D) $-\frac{1}{2}$

E) $\frac{1}{2}$

Q
4

If $0 \leq \theta < 2\pi$, $\cos\theta = \frac{4}{5}$ and $\sin\theta = -\frac{3}{5}$, then $\sec\left(\frac{\theta}{2}\right) =$

A) $\frac{\sqrt{10}}{3}$

B) $\frac{\sqrt{10}}{4}$

C) $-\frac{\sqrt{10}}{3}$

D) $-\frac{\sqrt{10}}{5}$

E) $-\frac{\sqrt{10}}{4}$

Q
5

If $\tan x = -\frac{\sqrt{5}}{2}, \frac{3\pi}{2} < x < 2\pi$, then $\sin\frac{x}{2} =$

A) $\frac{\sqrt{6}}{6}$

B) $-\frac{\sqrt{6}}{6}$

C) $\sqrt{6}$

D) $-\sqrt{6}$

E) $2\sqrt{6}$

Q
6

If $\tan\theta = -\frac{3}{4}, \frac{\pi}{2} < \theta < \pi$, then $\cos\left(\frac{\pi}{2} - \frac{\theta}{2}\right) =$

A) $\frac{2\sqrt{5}}{5}$

B) $-\frac{7}{25}$

C) $\frac{\sqrt{10}}{10}$

D) $-\frac{\sqrt{10}}{10}$

E) $\frac{3\sqrt{10}}{10}$

Q
7

If $\sin\theta = -\frac{3}{5}$ and $\tan\theta > 0$, then $\tan\frac{\theta}{2} =$

A) -3

B) $-\frac{3}{2}$

C) $-\frac{4}{3}$

D) $-\frac{1}{3}$

E) $-\frac{3}{4}$

Q
8

If $\sin\theta = -\frac{4}{5}$, with $180° < \theta < 270°$, then $\cos\frac{\theta}{2} + \sin 2\theta =$

A) $\frac{24-5\sqrt{5}}{25}$

B) $\frac{5\sqrt{5}-24}{25}$

C) $-\frac{24+5\sqrt{5}}{25}$

D) $\frac{24+5\sqrt{5}}{25}$

E) $\frac{\sqrt{5}-8}{5}$

Topic 2 | Exact Values of Special Angles

Evaluating half-angle, double-angle and product expressions at special angles • Questions 9–23

Q **9**

The exact value of $\tan(607.5°)$ is

A) $\sqrt{2}+1$

B) $-\sqrt{3}-3$

C) $-\sqrt{2}+1$

D) $-\sqrt{2}-1$

E) $\sqrt{2}-1$

Q **10**

$$(\sin 22.5° + \cos 22.5°)^2 = \boxtimes$$

A) $\frac{2+\sqrt{2}}{2}$

B) 1

C) $\frac{\sqrt{2}}{2}$

D) $\frac{2-\sqrt{2}}{2}$

E) 2

Q
11

By using the half-angle identities, $\cos\frac{3\pi}{8}$ is equal to

A) $\frac{\sqrt{2-\sqrt{2}}}{2}$

B) $-\frac{\sqrt{2+\sqrt{3}}}{2}$

C) $\frac{\sqrt{2+\sqrt{3}}}{2}$

D) $-\frac{\sqrt{2+\sqrt{2}}}{2}$

E) $\frac{\sqrt{2+\sqrt{2}}}{2}$

Q
12

The exact value of $2\sin(382.5°)$ is

A) $\sqrt{2-\sqrt{2}}$

B) $-\sqrt{2-\sqrt{2}}$

C) $-\sqrt{2+\sqrt{2}}$

D) $\sqrt{2+\sqrt{2}}$

E) $\sqrt{\sqrt{2}-2}$

Q
13

The exact value of $\tan(247.5°)$ is

A) $\sqrt{2}+1$

B) $\sqrt{2}-1$

C) $-\sqrt{2}+1$

D) $-\sqrt{2}-1$

E) $\sqrt{2}+2$

Q
14

$$\frac{1}{4}-\frac{1}{2}\sin^2 67.5° = \square$$

A) $-\frac{\sqrt{2}}{8}$

B) $-\frac{\sqrt{3}}{8}$

C) $\frac{\sqrt{3}}{8}$

D) $\frac{\sqrt{2}}{8}$

E) $\frac{\sqrt{6}}{8}$

Q
15

The exact value of $\sin 15° \left(8 - 16 \sin^2(7.5°)\right)$ is equal to

A) 2

B) 4

C) 8

D) $\frac{1}{2}$

E) $\frac{1}{4}$

Q
16

$$\sqrt{\frac{1 + \cos 110°}{2}} = \square$$

A) $\sin 35°$

B) $\cos 65°$

C) $\sin 15°$

D) $-\cos 55°$

E) $\tan 55°$

Q
17

The value of $\frac{4\sin 15^\circ \cos 15^\circ}{(\cos 15^\circ + \sin 15^\circ)(\cos 15^\circ - \sin 15^\circ)}$ is

A) $2\sqrt{3}+1$

B) $2\sqrt{2}$

C) $\sqrt{2}$

D) $2\sqrt{3}$

E) $\frac{2\sqrt{3}}{3}$

Q
18

$$\sin\frac{9\pi}{8}\cos\frac{\pi}{8} = \boxed{}$$

A) $-\frac{\sqrt{2}}{4}$

B) $-\frac{\sqrt{2}}{2}$

C) $\frac{\sqrt{2}}{8}$

D) $\frac{\sqrt{2}}{4}$

E) $\frac{\sqrt{2}}{2}$

Q
19

$(\sin 15° + \cos 15°)^2 + \frac{2\tan\frac{\pi}{3}}{1-\tan^2\frac{\pi}{3}}$ is equal to

A) $\frac{3-2\sqrt{3}}{2}$

B) $\frac{3+\sqrt{3}}{2}$

C) $\frac{3+2\sqrt{3}}{2}$

D) $-\frac{3+2\sqrt{3}}{2}$

E) $\frac{3-\sqrt{3}}{2}$

Q
20

$$3 - 8\sin^2(22.5°)\cos^2(22.5°) = ☒$$

A) 2

B) 0

C) 1

D) -1

E) -2

Q
21

$$\frac{\cot\frac{\pi}{12}-\tan\frac{\pi}{12}}{\cot\frac{\pi}{12}+\tan\frac{\pi}{12}}=\boxtimes$$

A) $\frac{\sqrt{3}}{2}$

B) $\frac{1}{2}$

C) $\frac{\sqrt{3}}{3}$

D) $\sqrt{3}$

E) $\frac{\sqrt{3}}{4}$

Q
22

$$\frac{1}{2}\sin 15^\circ \sin 75^\circ =\boxtimes$$

A) $\frac{1}{8}$

B) $-\frac{1}{4}$

C) $-\frac{\sqrt{3}}{2}$

D) $\frac{\sqrt{3}}{4}$

E) $\frac{\sqrt{3}}{8}$

Q
23

$$\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)=$$

A) 1

B) $\frac{1}{8}$

C) $\frac{3}{8}$

D) $\frac{3}{4}$

E) $\frac{1}{4}$

Topic 3 | Double-Angle Values and Inverse Trigonometric Functions

Evaluating cosine, tangent and cotangent of a double angle that contains an inverse function, and sin 2x + cos 2x from tan x • Questions 24–31

Q
24

$$\cos\left(\frac{1}{2}\tan^{-1}\frac{3}{4}\right) = ?$$

A) $\frac{3\sqrt{10}}{10}$

B) $\frac{2}{5}$

C) $\frac{3}{10}$

D) $\frac{\sqrt{10}}{10}$

E) $\frac{3}{5}$

Q
25

$$\cos\left(2\sin^{-1}\left(\frac{1}{4}\right)\right) = ?$$

A) $\frac{7}{8}$

B) $\frac{8}{9}$

C) $\frac{5}{8}$

D) $\frac{3}{8}$

E) $\frac{1}{8}$

Q
26

$$9\sqrt{2}\sin\left(2\cos^{-1}\left(-\frac{1}{3}\right)\right) = ?$$

A) -4

B) -6

C) 8

D) 6

E) -8

Q
27

$$\cot\left[2\cos^{-1}\left(-\frac{4}{5}\right)\right] = ?$$

A) $-\frac{7}{24}$

B) $\frac{24}{7}$

C) $\frac{5}{6}$

D) $-\frac{5}{12}$

E) $-\frac{7}{12}$

Q
28

The exact value of $\tan\left[2\cos^{-1}\frac{1}{4}\right]$ is

A) $-\frac{\sqrt{15}}{7}$

B) $\frac{\sqrt{15}}{4}$

C) $-\frac{\sqrt{13}}{7}$

D) $\frac{\sqrt{17}}{7}$

E) 34

Q
29

If $2\sin^{-1}\frac{3}{5} = \cos^{-1} x$, then $x =$

A) $\frac{7}{25}$

B) $-\frac{8}{5}$

C) $\frac{24}{25}$

D) $-\frac{13}{25}$

E) $-\frac{4}{5}$

Q
30

$$\tan\left(2\sin^{-1}\frac{2}{\sqrt{13}}\right) = ?$$

A) $\frac{12}{5}$

B) $\frac{4}{3}$

C) $\frac{5}{12}$

D) $\frac{4}{5}$

E) $\frac{3}{5}$

Q
31

If $\tan x = 3$ and $\sin x < 0$, then $\sin 2x + \cos 2x = ?$

A) $-\frac{1}{5}$

B) $\frac{7}{5}$

C) $\frac{6}{5}$

D) $-\frac{4}{5}$

E) 1

Topic 4 | Simplifying with Double-Angle and Half-Angle Identities

Rewriting expressions with double-angle and half-angle identities, and proving identities • Questions 32–43

Q
32

$$\cot\frac{x}{2} - \cos x \cot\frac{x}{2} = \boxtimes$$

A) $\sin x$

B) $\cos x$

C) $\tan x$

D) $\cot x$

E) $\csc x$

Q
33

$$\sin^2\frac{x}{2} = \boxtimes$$

A) $\frac{\tan x + \sin x}{\tan x}$

B) $\frac{1 - \tan x \sin x}{2\tan x}$

C) $\frac{2\tan x - \sin x}{2\tan x}$

D) $\frac{2\tan x}{\tan x + \sin x}$

E) $\frac{\tan x - \sin x}{2\tan x}$

Q
34

$\frac{1-\cos 3x}{2}$ is identical to:

A) $\sin^2 \frac{3x}{2}$

B) $\cos \frac{3x}{2}$

C) $\frac{1}{2} \sin \frac{3x}{2}$

D) $\cos^2 \frac{3x}{2}$

E) $\sin^2 6x$

Q
35

$$\sqrt{\frac{1-\sin \frac{17\pi}{5}}{2}} = ?$$

A) $\sin \frac{9\pi}{20}$

B) $\cos \frac{11\pi}{20}$

C) $-\sin \frac{\pi}{10}$

D) $-\cos \frac{\pi}{10}$

E) $\cos \frac{\pi}{10}$

Q
36

$$\frac{2\tan x\cos^2 x-\tan x}{1-\tan^2 x}=\square$$

A) $\sin x\cos x$

B) $\cos^2 x$

C) $-\cot x\sin^2 x$

D) $-\cot x\cos^2 x$

E) $-\sec x\csc^2 x$

Q
37

The expression $\frac{\sin 2x-\sin x}{2\cos^2 x+\cos x-1}$ simplifies to

A) $\tan\frac{x}{2}$

B) $\cot\frac{x}{2}$

C) $\cos\frac{x}{2}$

D) $\sin\frac{x}{2}$

E) $\sec\frac{x}{2}$

Q
38

$$\frac{4\tan x}{1+\tan^2 x} = \square$$

A) $2\sin 2x$

B) $2\cos 2x$

C) $2\cot 2x$

D) $2\sec 2x$

E) $2\csc 2x$

Q
39

$$\frac{\tan x}{\tan x+\cot x} - \frac{\cot x}{\cot x+\tan x} = \square$$

A) $-\cos 2x$

B) $\sin 2x$

C) $-\tan 2x$

D) $\cot 2x$

E) $-\sec 2x$

Q
40

$$\frac{\tan\theta + \cot\theta}{\tan\theta - \cot\theta} = ☒$$

A) $-\sec 2\theta$

B) $-\csc 2\theta$

C) $\csc 2\theta$

D) $\cos 2\theta$

E) $\sec 2\theta$

Q
41

$$\frac{\csc^2 x + \csc^4 x}{2 + \csc^2 x - \csc^4 x} = ☒$$

A) $-\sec 2x$

B) $\sec 2x$

C) $\csc 2x$

D) $-\csc 2x$

E) $\cot 2x$

Q
42

$$\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} - \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \square$$

A) $2\tan 2\theta$

B) $2\cot 2\theta$

C) $2\sec 2\theta$

D) $2\csc 2\theta$

E) $2\sin 2\theta$

Q
43

Which one of the following equations is an identity?

A) $\sin^4 x - \cos^4 x = -\cos 2x$

B) $\sin\frac{x}{2}\cos\frac{x}{2} = 2\sin x$

C) $\tan^2 x - \sec^2 x = 1$

D) $\sec(x) + \sec(-x) = 0$

E) $\sin x = \sqrt{1 - \cos^2 x}$

Topic 5 | Constants, Periods and Values from a Given Double Angle

Finding constants, amplitude and period, or a value when a double angle is given • Questions 44–53

Q **44**

If $\cos(2\alpha) = \frac{7}{25}$, $0 < \alpha < \frac{\pi}{2}$ and $\sin(2\beta) = -1$, $\frac{\pi}{2} < \beta < \pi$, then $\tan(\alpha - \beta) =$☒

A) 7

B) $\frac{1}{5}$

C) $-\frac{2}{5}$

D) 10

E) 5

Q **45**

If $\cos 2\,\theta = \frac{1}{2}$, where $\pi < \theta < \frac{3\pi}{2}$, then $\cot\theta =$☒

A) $\sqrt{3}$

B) $-\sqrt{2}$

C) $\frac{\sqrt{3}}{2}$

D) $\frac{\sqrt{2}}{3}$

E) $3\sqrt{2}$

Q
46

If $\left(\sin^2\frac{x}{4}\right)\left(\cos^2\frac{x}{4}\right) = a + b\cos x$, then $a \cdot b =$

A) $-\frac{1}{64}$

B) $-\frac{1}{36}$

C) $-\frac{1}{32}$

D) $-\frac{1}{48}$

E) $-\frac{1}{24}$

Q
47

If $\cos 2\theta = \frac{7}{25}$ and $\frac{\pi}{2} < \theta < \pi$, then $\tan\theta =$

A) $-\frac{3}{4}$

B) $\frac{3}{4}$

C) $-\frac{4}{3}$

D) $-\frac{24}{25}$

E) $\frac{4}{3}$

Q
48

If $\sin 3x = A\sin^3 x + B\sin x$, then $A + B =$☒

A) -7

B) -1

C) 1

D) -2

E) 2

Q
49

$\cos 3\alpha =$☒

A) $3\cos^3\alpha - \cos\alpha$

B) $2\cos^3\alpha - 4\cos\alpha$

C) $4\cos^3\alpha - 3\cos\alpha$

D) $3\cos^3\alpha + 4\cos\alpha$

E) $4\cos^3\alpha + \cos\alpha$

Q
50

The value of $1 - \cos^2(20°) - \cos^2(70°)$ is

A) $\cos^2(90°)$

B) $\sin^2(90°)$

C) $\sin^2(70°)$

D) $1 - \sin^2(20°)$

E) $\sin^2(20°) - \cos^2(20°)$

Q
51

If P is the period and A is the amplitude of the function $y = 2 \sin \pi\ x \cos \pi\ x$, then $A + P =$

A) 2

B) 0

C) 3

D) $\frac{3}{2}$

E) $1 + \sqrt{5}$

Q
52

If $p = \sin 165° \cos 165°$ and $q = \cos^2 \frac{\pi}{8} - \frac{1}{2}$, then $p + q =$

A) $\frac{\sqrt{2}-1}{4}$

B) $\frac{\sqrt{2}-1}{2}$

C) $\frac{\sqrt{2}}{4}$

D) $\frac{\sqrt{2}+1}{2}$

E) $\frac{1-\sqrt{2}}{2}$

Q
53

If $\sin x = -\frac{3}{5}$, $\pi < x < \frac{3\pi}{2}$, then $48 \cot 2x + 7 \sec 2x =$

A) 39

B) -39

C) 11

D) -11

E) 32

Answer Key

"Original" shows the question number in the source worksheet.

Q	Original	Ans.	Value
1	1	**A**	$-\frac{\sqrt{5}}{2}$
2	2	**A**	$\frac{1}{3}$
3	3	**A**	$-\frac{\sqrt{5}}{2}$
4	7	**C**	$-\frac{\sqrt{10}}{3}$
5	8	**A**	$\frac{\sqrt{6}}{6}$
6	20	**E**	$\frac{3\sqrt{10}}{10}$
7	21	**A**	-3
8	26	**A**	$\frac{24-5\sqrt{5}}{25}$
9	5	**A**	$\sqrt{2}+1$
10	10	**A**	$\frac{2+\sqrt{2}}{2}$
11	12	**A**	$\frac{\sqrt{2-\sqrt{2}}}{2}$
12	18	**A**	$\sqrt{2-\sqrt{2}}$
13	19	**A**	$\sqrt{2}+1$
14	27	**A**	$-\frac{\sqrt{2}}{8}$
15	28	**A**	2
16	30	**A**	$\sin 35°$
17	31	**E**	$\frac{2\sqrt{3}}{3}$
18	44	**A**	$-\frac{\sqrt{2}}{4}$
19	47	**A**	$\frac{3-2\sqrt{3}}{2}$
20	48	**A**	2
21	49	**A**	$\frac{\sqrt{3}}{2}$
22	53	**A**	$\frac{1}{8}$
23	23	**B**	$\frac{1}{8}$
24	11	**A**	$\frac{3\sqrt{10}}{10}$
25	33	**A**	$\frac{7}{8}$

Q	Original	Ans.	Value
26	34	E	-8
27	36	A	$-\frac{7}{24}$
28	39	A	$-\frac{\sqrt{15}}{7}$
29	42	A	$\frac{7}{25}$
30	51	A	$\frac{12}{5}$
31	52	A	$-\frac{1}{5}$
32	4	A	$\sin x$
33	14	E	$\frac{\tan x - \sin x}{2 \tan x}$
34	16	A	$\sin^2 \frac{3x}{2}$
35	17	A	$\sin \frac{9\pi}{20}$
36	23	A	$\sin x \cos x$
37	24	A	$\tan \frac{x}{2}$
38	25	A	$2 \sin 2x$
39	32	A	$-\cos 2x$
40	43	A	$-\sec 2\theta$
41	46	A	$-\sec 2x$
42	50	A	$2 \tan 2\theta$
43	59	A	$\sin^4 x - \cos^4 x = -\cos 2x$
44	6	A	7
45	13	A	$\sqrt{3}$
46	29	A	$-\frac{1}{64}$
47	35	A	$-\frac{3}{4}$
48	37	B	-1
49	38	C	$4 \cos^3 \alpha - 3 \cos \alpha$
50	41	A	$\cos^2(90°)$
51	45	A	2
52	54	A	$\frac{\sqrt{2}-1}{4}$
53	55	A	39