7.3: (Double-Angle, Half-Angle)

If $\tan\alpha = -\frac{4}{3}, \frac{3\pi}{2} < \alpha < 2\pi$, then $\sec\frac{\alpha}{2} =$

(a) $-\frac{\sqrt{5}}{2}$

(b) $\frac{2}{\sqrt{3}}$

(c) $\frac{2}{\sqrt{3}}$

(d) $-\frac{\sqrt{3}}{2}$

(e) $-\sqrt{5}$

If $\sin\theta = \frac{2\sqrt{2}}{3}, 0 < \theta < \frac{\pi}{2}$, then $\sin^2\frac{\theta}{2} =$

A) $\frac{1}{3}$

B) $\frac{1}{2}$

C) $\frac{2}{3}$

D) 3

E) 2

If $\csc\theta = -\frac{5}{4}$, where $\frac{3\pi}{2} < \theta < 2\pi$, then $\sec\frac{\theta}{2} =$

A) $-\frac{\sqrt{5}}{2}$

B) $-\frac{\sqrt{5}}{5}$

C) $\frac{\sqrt{5}}{5}$

D) $-\frac{1}{2}$

E) $\frac{1}{2}$

$\cot\frac{x}{2} - \cos x\cot\frac{x}{2} =$

(a) $\sin x$

(b) $\cos x$

(c) $\tan x$

(d) $\cot x$

(e) $\csc x$

The exact value of $\tan(607.5°)$, is

A) $\sqrt{2}+1$

B) $-\sqrt{3}-3$

C) $-\sqrt{2}+1$

D) $-\sqrt{2}-1$

E) $\sqrt{2}-1$

If $\cos(2\alpha)=\frac{7}{25}, 0<\alpha<\frac{\pi}{2}$ and $\sin(2\beta)=-1,\ \frac{\pi}{2}<\beta<\pi$, then $\tan(\alpha-\beta)=$

A) 7

B) $\frac{1}{5}$

C) $-\frac{2}{5}$

D) 10

E) 5

If $0 \leq \theta < 2\pi, \cos\theta = \frac{4}{5}$ and $\sin\theta = \frac{-3}{5}$, then $\sec\left(\frac{\theta}{2}\right) =$

A) $\frac{\sqrt{10}}{3}$

B) $\frac{\sqrt{10}}{4}$

C) $-\frac{\sqrt{10}}{3}$

D) $-\frac{\sqrt{10}}{5}$

E) $-\frac{\sqrt{10}}{4}$

If $\tan x = -\frac{\sqrt{5}}{2}, \frac{3\pi}{2} < x < 2\pi$, then $\sin\frac{x}{2} =$

A) $\frac{\sqrt{6}}{6}$

B) $-\frac{\sqrt{6}}{6}$

C) $\sqrt{6}$

D) $-\sqrt{6}$

E) $2\sqrt{6}$

If $\csc\theta = -\frac{5}{4}$, where $\frac{3\pi}{2} < \theta < 2\pi$, then $\sec\frac{\theta}{2} =$

A) $-\frac{\sqrt{5}}{2}$

B) $-\frac{\sqrt{5}}{5}$

C) $\frac{\sqrt{5}}{5}$

D) $-\frac{1}{2}$

E) $\frac{1}{2}$

$(\sin 22.5° + \cos 22.5°)^2 =$

A) $\frac{2+\sqrt{2}}{2}$

B) 1

C) $\frac{\sqrt{2}}{2}$

D) $\frac{2-\sqrt{2}}{2}$

E) 2

$\cos\left(\frac{1}{2}\tan^{-1}\frac{3}{4}\right) =$

A) $\frac{3\sqrt{10}}{10}$

B) $\frac{2}{5}$

C) $\frac{3}{10}$

D) $\frac{\sqrt{10}}{10}$

E) $\frac{3}{5}$

By using the half angle identities $\cos\frac{3\pi}{8}$ is equal to

A) $\frac{\sqrt{2-\sqrt{2}}}{2}$

B) $-\frac{\sqrt{2+\sqrt{3}}}{2}$

C) $\frac{\sqrt{2+\sqrt{3}}}{2}$

D) $-\frac{\sqrt{2\cdot\sqrt{2}}}{2}$

E) $\frac{\sqrt{2\cdot\sqrt{2}}}{2}$

If $\cos 2\theta = \frac{1}{2}$, where $\pi < \theta < \frac{3\pi}{2}$, then $\cot\theta =$

A) $\sqrt{3}$

B) $-\sqrt{2}$

C) $\frac{\sqrt{3}}{2}$

D) $\frac{\sqrt{2}}{3}$

E) $3\sqrt{2}$

$\sin^2\frac{x}{2} =$

A) $\frac{\tan x+\sin x}{\tan x}$

B) $\frac{1-\tan x\sin x}{2\tan x}$

C) $\frac{2\tan x-\sin x}{2\tan x}$

D) $\frac{2\tan x}{\tan x+\sin x}$

E) $\frac{\tan x-\sin x}{2\tan x}$

If $0 \leq \theta < 2\pi, \cos\theta = \frac{4}{5}$ and $\sin\theta = \frac{-3}{5}$, then $\sec\left(\frac{\theta}{2}\right) =$

A) $\frac{\sqrt{10}}{3}$

B) $\frac{\sqrt{10}}{4}$

C) $-\frac{\sqrt{10}}{3}$

D) $-\frac{\sqrt{10}}{5}$

E) $-\frac{\sqrt{10}}{4}$

$\frac{1-\cos 3x}{2}$ is identical to:

A) $\sin^2\frac{3x}{2}$

B) $\cos\frac{3x}{2}$

C) $\frac{1}{2}\sin\frac{3x}{2}$

D) $\cos^2\frac{3x}{2}$

E) $\sin^2 6x$

$$\sqrt{\frac{1-\sin\frac{17\pi}{5}}{2}} =$$

A) $\sin\frac{9\pi}{20}$

B) $\cos\frac{11\pi}{20}$

C) $-\sin\frac{\pi}{10}$

D) $-\cos\frac{\pi}{10}$

E) $\cos\frac{\pi}{10}$

The exact value of $2\sin(382.5°) =$

A) $\sqrt{2-\sqrt{2}}$

B) $-\sqrt{2-\sqrt{2}}$

C) $-\sqrt{2+\sqrt{2}}$

D) $\sqrt{2+\sqrt{2}}$

E) $\sqrt{\sqrt{2}-2}$

The exact value of $\tan(247.5°)$ is

A) $\sqrt{2}+1$

B) $\sqrt{2}-1$

C) $-\sqrt{2}+1$

D) $-\sqrt{2}-1$

E) $\sqrt{2}+2$

If $\tan\theta = -\frac{3}{4}, \frac{\pi}{2} < \theta < \pi$, then $\cos\left(\frac{\pi}{2} - \frac{\theta}{2}\right) =$

A) $\frac{2\sqrt{5}}{5}$

B) $-\frac{7}{25}$

C) $\frac{\sqrt{10}}{10}$

D) $-\frac{\sqrt{10}}{10}$

E) $\frac{3\sqrt{10}}{10}$

If $\sin\theta = -\frac{3}{5}$ and $\tan\theta > 0$, then $\tan\frac{\theta}{2} =$

A) -3

B) $-\frac{3}{2}$

C) $-\frac{4}{3}$

D) $-\frac{1}{3}$

E) $-\frac{3}{4}$

$\cos\left(\frac{1}{2}\tan^{-1}\frac{3}{4}\right) =$

A) $\frac{3\sqrt{10}}{10}$

B) $\frac{2}{5}$

C) $\frac{3}{10}$

D) $\frac{\sqrt{10}}{10}$

E) $\frac{3}{5}$

$$\frac{2\tan x\cos^2 x - \tan x}{1 - \tan^2 x} =$$

A) $\sin x\cos x$

B) $\cos^2 x$

C) $-\cot x\sin^2 x$

D) $\cot x\sin^2 x$

E) $-\sec x\csc^2 x$

The expression $\frac{\sin 2x - \sin x}{2\cos^2 x + \cos x - 1}$ simplifies to

(a) $\tan\frac{x}{2}$

(b) $\cot\frac{x}{2}$

(c) $\cos\frac{x}{2}$

(d) $\sin\frac{x}{2}$

(e) $\sec\frac{x}{2}$

$$\frac{4\tan x}{1+\tan^2 x} =$$

(a) $2\sin 2x$

(b) $2\cos 2x$

(c) $2\cot 2x$

(d) $2\sec 2x$

(e) $2\csc 2x$

If $\sin\theta = -\frac{4}{5}$, with $180° < \theta < 270°$, then $\cos\frac{\theta}{2} + \sin 2\theta =$

A) $\frac{24-5\sqrt{5}}{25}$

B) $\frac{5\sqrt{5}-24}{25}$

C) $-\frac{24+5\sqrt{5}}{25}$

D) $\frac{24+5\sqrt{5}}{25}$

E) $\frac{\sqrt{5}-8}{5}$

$\frac{1}{4} - \frac{1}{2}\sin^2 67.5° =$

A) $-\frac{\sqrt{2}}{8}$

B) $-\frac{\sqrt{3}}{8}$

C) $\frac{\sqrt{3}}{8}$

D) $\frac{\sqrt{2}}{8}$

E) $\frac{\sqrt{6}}{8}$

The exact value of $\sin 15°\left(8 - 16\sin^2(7.5°)\right)$ is equal to

(a) 2

(b) 4

(c) 8

(d) $\frac{1}{2}$

(e) $\frac{1}{4}$

If $\left(\sin^2\frac{x}{4}\right)\left(\cos^2\frac{x}{4}\right) = a + b\cos x$, then $a \cdot b =$

A) $-\frac{1}{64}$

B) $-\frac{1}{36}$

C) $-\frac{1}{32}$

D) $-\frac{1}{48}$

E) $-\frac{1}{24}$

$\sqrt{\frac{1+\cos 110^\circ}{2}} =$

(a) $\sin 35^\circ$

(b) $\cos 65^\circ$

(c) $\sin 15^\circ$

(d) $-\cos 55^\circ$

(e) $\tan 55^\circ$

The value of $\frac{4\sin 15^\circ \cos 15^\circ}{(\cos 15^\circ + \sin 15^\circ)(\cos 15^\circ - \sin 15^\circ)}$ is

A) $2\sqrt{3}+1$

B) $2\sqrt{2}$

C) $\sqrt{2}$

D) $2\sqrt{3}$

E) $\frac{2\sqrt{3}}{3}$

$$\frac{\tan x}{\tan x + \cot x} - \frac{\cot x}{\cot x + \tan x} =$$

A) $-\cos 2x$

B) $\sin 2x$

C) $-\tan 2x$

D) $\cot 2x$

E) $-\sec 2x$

$$\cos\left(2\sin^{-1}\left(\frac{1}{4}\right)\right) =$$

A) $\frac{7}{8}$

B) $\frac{8}{9}$

C) $\frac{5}{8}$

D) $\frac{3}{8}$

E) $\frac{1}{8}$

$$9\sqrt{2}\sin\left(2\cos^{-1}\left(-\frac{1}{3}\right)\right) =$$

A) -4

B) -6

C) 8

D) 6

E) -8

If $\cos 2\theta = \frac{7}{25}$ and $\frac{\pi}{2} < \theta < \pi$, then $\tan\theta =$

A) $-\frac{3}{4}$

B) $\frac{3}{4}$

C) $-\frac{4}{3}$

D) $-\frac{24}{25}$

E) $\frac{4}{3}$

$\cot\left[2\cos^{-1}\left(-\frac{4}{5}\right)\right] =$

A) $-\frac{7}{24}$

B) $\frac{24}{7}$

C) $\frac{5}{6}$

D) $-\frac{5}{12}$

E) $-\frac{7}{12}$

If $\sin 3x = A\sin^3 x + B\sin x$, then $A + B =$

A) -7

B) -1

C) 1

D) -2

E) 2

$\cos 3\alpha =$

A) $3\cos^3 \alpha - \cos \alpha$

B) $2\cos^3 \alpha - 4\cos \alpha$

C) $4\cos^3 \alpha - 3\cos \alpha$

D) $3\cos^3 \alpha + 4\cos \alpha$

E) $4\cos^3 \alpha + \cos \alpha$

The exact value of $\tan\left[2\cos^{-1}\frac{1}{4}\right] =$

A) $-\frac{\sqrt{15}}{7}$

B) $\frac{\sqrt{15}}{4}$

C) $-\frac{\sqrt{13}}{7}$

D) $\frac{\sqrt{17}}{7}$

E) 34

$\cot\left[2\cos^{-1}\left(-\frac{4}{5}\right)\right] =$

A) $-\frac{7}{24}$

B) $-\frac{24}{25}$

C) $-\frac{5}{12}$

D) $\frac{24}{7}$

E) $\frac{7}{24}$

The value of $1 - \cos^2(20°) - \cos^2(70°)$ is

A) $\cos^2(90°)$

B) $\sin^2(90°)$

C) $\sin^2(70°)$

D) $1 - \sin^2(20°)$

E) $\sin^2(20°) - \cos^2(20°)$

If $2\sin^{-1}\frac{3}{5} = 2\pi + \cos^{-1} x$, then $x =$

A) $\frac{7}{25}$

B) $-\frac{8}{5}$

C) $\frac{24}{25}$

D) $-\frac{13}{25}$

E) $-\frac{4}{5}$

$$\frac{\tan\theta + \cot\theta}{\tan\theta - \cot\theta} =$$

A) $-\sec 2\theta$

B) $-\csc 2\theta$

C) $\csc 2\theta$

D) $\cos 2\theta$

E) $\sec 2\theta$

$$\sin\frac{9\pi}{8}\cos\frac{\pi}{8} =$$

A) $-\frac{\sqrt{2}}{4}$

B) $-\frac{\sqrt{2}}{2}$

C) $\frac{\sqrt{2}}{8}$

D) $\frac{\sqrt{2}}{4}$

E) $\frac{\sqrt{2}}{2}$

If P is the period and A is the amplitude of the function $y = 2\sin \pi x \cos \pi x$, then $\mathrm{A} + \mathrm{P} =$

A) 2

B) 0

C) 3

D) $\frac{3}{2}$

E) $1 + \sqrt{5}$

$$\frac{\csc^2 x + \csc^4 x}{2 + \csc^2 x - \csc^4 x} =$$

A) $-\sec 2x$

B) $\sec 2x$

C) $\csc 2x$

D) $-\csc 2x$

E) $\cot 2x$

$(\sin^\circ 15 + \cos^\circ 15)^2 + \frac{2\tan\frac{\pi}{3}}{1-\tan^2\frac{\pi}{3}}$ equals to

A) $\frac{3-2\sqrt{3}}{2}$

B) $\frac{3+\sqrt{3}}{2}$

C) $\frac{3+2\sqrt{3}}{2}$

D) $-\frac{3+2\sqrt{3}}{2}$

E) $\frac{3-\sqrt{3}}{2}$

$3 - 8\sin^2(22.5^\circ)\cos^2(22.5^\circ) =$

A) 2

B) 0

C) 1

D) -1

E) -2

$$\frac{\cot\frac{\pi}{12}-\tan\frac{\pi}{12}}{\cot\frac{\pi}{12}+\tan\frac{\pi}{12}}=$$

A) $\frac{\sqrt{3}}{2}$

B) $\frac{1}{2}$

C) $\frac{\sqrt{3}}{3}$

D) $\sqrt{3}$

E) $\frac{\sqrt{3}}{4}$

$$\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}-\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}=$$

A) $2\tan 2\theta$

B) $2\cot 2\theta$

C) $2\sec 2\theta$

D) $2\csc 2\theta$

E) $2\sin 2\theta$

$\tan\left(2\sin^{-1}\frac{2}{\sqrt{13}}\right) =$

A) $\frac{12}{5}$

B) $\frac{4}{3}$

C) $\frac{5}{12}$

D) $\frac{4}{5}$

E) $\frac{3}{5}$

If $\tan x = 3, \sin x < 0$, then $\sin 2x + \cos 2x =$

A) $-\frac{1}{5}$

B) $\frac{7}{5}$

C) $\frac{6}{5}$

D) $-\frac{4}{5}$

E) 1

$\frac{1}{2}\sin 15°\sin 75° =$

A) $\frac{1}{8}$

B) $-\frac{1}{4}$

C) $-\frac{\sqrt{3}}{2}$

D) $\frac{\sqrt{3}}{4}$

E) $\frac{\sqrt{3}}{8}$

If $p = \sin 165°\cos 165°$ and $q = \cos^2\frac{\pi}{8} - \frac{1}{2}$ then $p + q =$

A) $\frac{\sqrt{2}-1}{4}$

B) $\frac{\sqrt{2}-1}{2}$

C) $\frac{\sqrt{2}}{4}$

D) $\frac{\sqrt{2}+1}{2}$

E) $\frac{1-\sqrt{2}}{2}$

If $\sin x = -\frac{3}{5}, \pi < x < \frac{3\pi}{2}$, then $48\cot 2x + 7\sec 2x =$

A) 39

B) -39

C) 11

D) -11

E) 32

If $\cos 2\theta = \frac{1}{2}$, where $\pi < \theta < \frac{3\pi}{2}$, then $\cot \theta =$

A) $\sqrt{3}$

B) $-\sqrt{2}$

C) $\frac{\sqrt{3}}{2}$

D) $\frac{\sqrt{2}}{3}$

E) $3\sqrt{2}$

The exact value of $\tan\left[2\cos^{-1}\frac{1}{4}\right] =$

A) $-\frac{\sqrt{15}}{7}$

B) $\frac{\sqrt{15}}{4}$

C) $-\frac{\sqrt{13}}{7}$

D) $\frac{\sqrt{17}}{7}$

E) 34

$\cot\left[2\cos^{-1}\left(-\frac{4}{5}\right)\right] =$

A) $-\frac{7}{24}$

B) $-\frac{24}{25}$

C) $-\frac{5}{12}$

D) $\frac{24}{7}$

E) $\frac{7}{24}$

Which one of the following equations is an identity?

A) $\sin^4 x - \cos^4 x = -\cos 2x$

B) $\sin\frac{x}{2}\cos\frac{x}{2} = 2\sin x.$

C) $\tan^2 x - \sec^2 x = 1$

D) $\sec(x) + \sec(-x) = 0$

E) $\sin x = \sqrt{1 - \cos^2 x}$