

### 7.3: (Double-Angle, Half-Angle)

If  $\tan \alpha = -\frac{4}{3}$ ,  $\frac{3\pi}{2} < \alpha < 2\pi$ , then  $\sec \frac{\alpha}{2} =$

(a)  $-\frac{\sqrt{5}}{2}$

(b)  $\frac{2}{\sqrt{3}}$

(c)  $\frac{2}{\sqrt{3}}$

(d)  $-\frac{\sqrt{3}}{2}$

(e)  $-\sqrt{5}$

Half Angle  
Identity.

If  $\sin \theta = \frac{2\sqrt{2}}{3}$ ,  $0 < \theta < \frac{\pi}{2}$ , then  $\sin^2 \frac{\theta}{2} =$

A)  $\frac{1}{3}$

B)  $\frac{1}{2}$

C)  $\frac{2}{3}$

D) 3

E) 2

Half Angle  
Identity.

If  $\csc \theta = -\frac{5}{4}$ , where  $\frac{3\pi}{2} < \theta < 2\pi$ , then  $\sec \frac{\theta}{2} =$

A)  $-\frac{\sqrt{5}}{2}$

B)  $-\frac{\sqrt{5}}{5}$

C)  $\frac{\sqrt{5}}{5}$

D)  $-\frac{1}{2}$

E)  $\frac{1}{2}$

Half Angle  
Identity.

$$\cot \frac{x}{2} - \cos x \cot \frac{x}{2} =$$

(a)  $\sin x$

(b)  $\cos x$

(c)  $\tan x$

(d)  $\cot x$

(e)  $\csc x$

Half Angle  
Identity.

The exact value of  $\tan(607.5^\circ)$ , is

A)  $\sqrt{2} + 1$

B)  $-\sqrt{3} - 3$

C)  $-\sqrt{2} + 1$

D)  $-\sqrt{2} - 1$

E)  $\sqrt{2} - 1$

Half Angle  
Identity.

If  $\cos(2\alpha) = \frac{7}{25}$ ,  $0 < \alpha < \frac{\pi}{2}$  and  $\sin(2\beta) = -1$ ,  $\frac{\pi}{2} < \beta < \pi$ , then  $\tan(\alpha - \beta) =$

A) 7

B)  $\frac{1}{5}$

C)  $-\frac{2}{5}$

D) 10

E) 5

Half Angle  
Identity.

If  $0 \leq \theta < 2\pi$ ,  $\cos \theta = \frac{4}{5}$  and  $\sin \theta = \frac{-3}{5}$ , then  $\sec\left(\frac{\theta}{2}\right) =$

A)  $\frac{\sqrt{10}}{3}$

B)  $\frac{\sqrt{10}}{4}$

C)  $-\frac{\sqrt{10}}{3}$

D)  $-\frac{\sqrt{10}}{5}$

E)  $-\frac{\sqrt{10}}{4}$

Half Angle  
Identity.

If  $\tan x = -\frac{\sqrt{5}}{2}$ ,  $\frac{3\pi}{2} < x < 2\pi$ , then  $\sin\frac{x}{2} =$

A)  $\frac{\sqrt{6}}{6}$

B)  $-\frac{\sqrt{6}}{6}$

C)  $\sqrt{6}$

D)  $-\sqrt{6}$

E)  $2\sqrt{6}$

Half Angle  
Identity.

If  $\csc \theta = -\frac{5}{4}$ , where  $\frac{3\pi}{2} < \theta < 2\pi$ , then  $\sec \frac{\theta}{2} =$

- A)  $-\frac{\sqrt{5}}{2}$
- B)  $-\frac{\sqrt{5}}{5}$
- C)  $\frac{\sqrt{5}}{5}$
- D)  $-\frac{1}{2}$
- E)  $\frac{1}{2}$

Half Angle  
Identity.

$$(\sin 22.5^\circ + \cos 22.5^\circ)^2 =$$

- A)  $\frac{2+\sqrt{2}}{2}$
- B) 1
- C)  $\frac{\sqrt{2}}{2}$
- D)  $\frac{2-\sqrt{2}}{2}$
- E) 2

Half Angle  
Identity.

$$\cos\left(\frac{1}{2}\tan^{-1}\frac{3}{4}\right) =$$

- A)  $\frac{3\sqrt{10}}{10}$
- B)  $\frac{2}{5}$
- C)  $\frac{3}{10}$
- D)  $\frac{\sqrt{10}}{10}$
- E)  $\frac{3}{5}$

Half Angle  
Identity.

By using the half angle identities  $\cos \frac{3\pi}{8}$  is equal to

A)  $\frac{\sqrt{2-\sqrt{2}}}{2}$

B)  $-\frac{\sqrt{2+\sqrt{3}}}{2}$

C)  $\frac{\sqrt{2+\sqrt{3}}}{2}$

D)  $-\frac{\sqrt{2\cdot\sqrt{2}}}{2}$

E)  $\frac{\sqrt{2\cdot\sqrt{2}}}{2}$

Half Angle  
Identity.

If  $\cos 2\theta = \frac{1}{2}$ , where  $\pi < \theta < \frac{3\pi}{2}$ , then  $\cot \theta =$

A)  $\sqrt{3}$

B)  $-\sqrt{2}$

C)  $\frac{\sqrt{3}}{2}$

D)  $\frac{\sqrt{2}}{3}$

E)  $3\sqrt{2}$

Half Angle  
Identity.

$$\sin^2 \frac{x}{2} =$$

A)  $\frac{\tan x + \sin x}{\tan x}$

B)  $\frac{1 - \tan x \sin x}{2 \tan x}$

C)  $\frac{2 \tan x - \sin x}{2 \tan x}$

D)  $\frac{2 \tan x}{\tan x + \sin x}$

E)  $\frac{\tan x - \sin x}{2 \tan x}$

Half Angle  
Identity.

If  $0 \leq \theta < 2\pi$ ,  $\cos \theta = \frac{4}{5}$  and  $\sin \theta = \frac{-3}{5}$ , then  $\sec\left(\frac{\theta}{2}\right) =$

A)  $\frac{\sqrt{10}}{3}$

B)  $\frac{\sqrt{10}}{4}$

C)  $-\frac{\sqrt{10}}{3}$

D)  $-\frac{\sqrt{10}}{5}$

E)  $-\frac{\sqrt{10}}{4}$

Half Angle  
Identity.

$\frac{1-\cos 3x}{2}$  is identical to:

A)  $\sin^2 \frac{3x}{2}$

B)  $\cos \frac{3x}{2}$

C)  $\frac{1}{2} \sin \frac{3x}{2}$

D)  $\cos^2 \frac{3x}{2}$

E)  $\sin^2 6x$

Half Angle  
Identity.

$$\sqrt{\frac{1 - \sin \frac{17\pi}{5}}{2}} =$$

A)  $\sin \frac{9\pi}{20}$

B)  $\cos \frac{11\pi}{20}$

C)  $-\sin \frac{\pi}{10}$

D)  $-\cos \frac{\pi}{10}$

E)  $\cos \frac{\pi}{10}$

Half Angle  
Identity.

The exact value of  $2\sin(382.5^\circ) =$

- A)  $\sqrt{2 - \sqrt{2}}$
- B)  $-\sqrt{2 - \sqrt{2}}$
- C)  $-\sqrt{2 + \sqrt{2}}$
- D)  $\sqrt{2 + \sqrt{2}}$
- E)  $\sqrt{\sqrt{2} - 2}$

Half Angle  
Identity.

The exact value of  $\tan(247.5^\circ)$  is

- A)  $\sqrt{2} + 1$
- B)  $\sqrt{2} - 1$
- C)  $-\sqrt{2} + 1$
- D)  $-\sqrt{2} - 1$
- E)  $\sqrt{2} + 2$

Half Angle  
Identity.

If  $\tan \theta = -\frac{3}{4}$ ,  $\frac{\pi}{2} < \theta < \pi$ , then  $\cos\left(\frac{\pi}{2} - \frac{\theta}{2}\right) =$

- A)  $\frac{2\sqrt{5}}{5}$
- B)  $-\frac{7}{25}$
- C)  $\frac{\sqrt{10}}{10}$
- D)  $-\frac{\sqrt{10}}{10}$
- E)  $\frac{3\sqrt{10}}{10}$

Half Angle  
Identity.

If  $\sin \theta = -\frac{3}{5}$  and  $\tan \theta > 0$ , then  $\tan \frac{\theta}{2} =$

A) -3

B)  $-\frac{3}{2}$

C)  $-\frac{4}{3}$

D)  $-\frac{1}{3}$

E)  $-\frac{3}{4}$

Half Angle  
Identity.

$$\cos\left(\frac{1}{2}\tan^{-1}\frac{3}{4}\right) =$$

A)  $\frac{3\sqrt{10}}{10}$

B)  $\frac{2}{5}$

C)  $\frac{3}{10}$

D)  $\frac{\sqrt{10}}{10}$

E)  $\frac{3}{5}$

Half Angle  
Identity.

$$\frac{2\tan x \cos^2 x - \tan x}{1 - \tan^2 x} =$$

A)  $\sin x \cos x$

B)  $\cos^2 x$

C)  $-\cot x \sin^2 x$

D)  $\cot x \sin^2 x$

E)  $-\sec x \csc^2 x$

Double Angle  
and Half Angle  
Identity.

The expression  $\frac{\sin 2x - \sin x}{2\cos^2 x + \cos x - 1}$  simplifies to

(a)  $\tan \frac{x}{2}$

(b)  $\cot \frac{x}{2}$

(c)  $\cos \frac{x}{2}$

(d)  $\sin \frac{x}{2}$

(e)  $\sec \frac{x}{2}$

Double Angle  
and Half Angle  
Identity.

$$\frac{4\tan x}{1 + \tan^2 x} =$$

(a)  $2\sin 2x$

(b)  $2\cos 2x$

(c)  $2\cot 2x$

(d)  $2\sec 2x$

(e)  $2\csc 2x$

Double Angle  
and Half Angle  
Identity.

If  $\sin \theta = -\frac{4}{5}$ , with  $180^\circ < \theta < 270^\circ$ , then  $\cos \frac{\theta}{2} + \sin 2\theta =$

A)  $\frac{24-5\sqrt{5}}{25}$

B)  $\frac{5\sqrt{5}-24}{25}$

C)  $-\frac{24+5\sqrt{5}}{25}$

D)  $\frac{24+5\sqrt{5}}{25}$

E)  $\frac{\sqrt{5}-8}{5}$

Double Angle  
and Half Angle  
Identity.

$$\frac{1}{4} - \frac{1}{2} \sin^2 67.5^\circ =$$

A)  $-\frac{\sqrt{2}}{8}$

B)  $-\frac{\sqrt{3}}{8}$

C)  $\frac{\sqrt{3}}{8}$

D)  $\frac{\sqrt{2}}{8}$

E)  $\frac{\sqrt{6}}{8}$

Double Angle  
and Half Angle  
Identity.

The exact value of  $\sin 15^\circ (8 - 16\sin^2(7.5^\circ))$  is equal to

(a) 2

(b) 4

(c) 8

(d)  $\frac{1}{2}$

(e)  $\frac{1}{4}$

Double Angle  
and Half Angle  
Identity.

If  $(\sin^2 \frac{x}{4})(\cos^2 \frac{x}{4}) = a + b\cos x$ , then  $a \cdot b =$

A)  $-\frac{1}{64}$

B)  $-\frac{1}{36}$

C)  $-\frac{1}{32}$

D)  $-\frac{1}{48}$

E)  $-\frac{1}{24}$

Double Angle  
and Half Angle  
Identity.

$$\sqrt{\frac{1+\cos 110^\circ}{2}} =$$

- (a)  $\sin 35^\circ$   
(b)  $\cos 65^\circ$   
(c)  $\sin 15^\circ$   
(d)  $-\cos 55^\circ$   
(e)  $\tan 55^\circ$

Double Angle  
and Half Angle  
Identity.

The value of  $\frac{4\sin 15^\circ \cos 15^\circ}{(\cos 15^\circ + \sin 15^\circ)(\cos 15^\circ - \sin 15^\circ)}$  is

- A)  $2\sqrt{3} + 1$   
B)  $2\sqrt{2}$   
C)  $\sqrt{2}$   
D)  $2\sqrt{3}$   
E)  $\frac{2\sqrt{3}}{3}$

Double Angle  
Identities.

$$\frac{\tan x}{\tan x + \cot x} - \frac{\cot x}{\cot x + \tan x} =$$

- A)  $-\cos 2x$   
B)  $\sin 2x$   
C)  $-\tan 2x$   
D)  $\cot 2x$   
E)  $-\sec 2x$

Double Angle  
Identities.

$$\cos\left(2\sin^{-1}\left(\frac{1}{4}\right)\right) =$$

A)  $\frac{7}{8}$

B)  $\frac{8}{9}$

C)  $\frac{5}{8}$

D)  $\frac{3}{8}$

E)  $\frac{1}{8}$

Double Angle  
Identities.

$$9\sqrt{2}\sin\left(2\cos^{-1}\left(-\frac{1}{3}\right)\right) =$$

A) -4

B) -6

C) 8

D) 6

E) -8

Double Angle  
Identities.

If  $\cos 2\theta = \frac{7}{25}$  and  $\frac{\pi}{2} < \theta < \pi$ , then  $\tan \theta =$

A)  $-\frac{3}{4}$

B)  $\frac{3}{4}$

C)  $-\frac{4}{3}$

D)  $-\frac{24}{25}$

E)  $\frac{4}{3}$

Double Angle  
Identities.

$$\cot \left[ 2\cos^{-1} \left( -\frac{4}{5} \right) \right] =$$

A)  $-\frac{7}{24}$

B)  $\frac{24}{7}$

C)  $\frac{5}{6}$

D)  $-\frac{5}{12}$

E)  $-\frac{7}{12}$

Double Angle  
Identities.

If  $\sin 3x = A\sin^3 x + B\sin x$ , then  $A + B =$

A) -7

B) -1

C) 1

D) -2

E) 2

Double Angle  
Identities.

$$\cos 3\alpha =$$

A)  $3\cos^3 \alpha - \cos \alpha$

B)  $2\cos^3 \alpha - 4\cos \alpha$

C)  $4\cos^3 \alpha - 3\cos \alpha$

D)  $3\cos^3 \alpha + 4\cos \alpha$

E)  $4\cos^3 \alpha + \cos \alpha$

Double Angle  
Identities.

The exact value of  $\tan \left[ 2\cos^{-1} \frac{1}{4} \right] =$

A)  $-\frac{\sqrt{15}}{7}$

B)  $\frac{\sqrt{15}}{4}$

C)  $-\frac{\sqrt{13}}{7}$

D)  $\frac{\sqrt{17}}{7}$

E) 34

Double Angle  
Identities.

$\cot \left[ 2\cos^{-1} \left( -\frac{4}{5} \right) \right] =$

A)  $-\frac{7}{24}$

B)  $-\frac{24}{25}$

C)  $-\frac{5}{12}$

D)  $\frac{24}{7}$

E)  $\frac{7}{24}$

Double Angle  
Identities.

The value of  $1 - \cos^2(20^\circ) - \cos^2(70^\circ)$  is

A)  $\cos^2(90^\circ)$

B)  $\sin^2(90^\circ)$

C)  $\sin^2(70^\circ)$

D)  $1 - \sin^2(20^\circ)$

E)  $\sin^2(20^\circ) - \cos^2(20^\circ)$

Double Angle  
Identities.

If  $2\sin^{-1}\frac{3}{5} = 2\pi + \cos^{-1} x$ , then  $x =$

A)  $\frac{7}{25}$

B)  $-\frac{8}{5}$

C)  $\frac{24}{25}$

D)  $-\frac{13}{25}$

E)  $-\frac{4}{5}$

Double Angle  
Identities.

$$\frac{\tan \theta + \cot \theta}{\tan \theta - \cot \theta} =$$

A)  $-\sec 2\theta$

B)  $-\csc 2\theta$

C)  $\csc 2\theta$

D)  $\cos 2\theta$

E)  $\sec 2\theta$

Double Angle  
Identities.

$$\sin \frac{9\pi}{8} \cos \frac{\pi}{8} =$$

A)  $-\frac{\sqrt{2}}{4}$

B)  $-\frac{\sqrt{2}}{2}$

C)  $\frac{\sqrt{2}}{8}$

D)  $\frac{\sqrt{2}}{4}$

E)  $\frac{\sqrt{2}}{2}$

Double Angle  
Identities.

If P is the period and A is the amplitude of the function  $y = 2\sin \pi x \cos \pi x$ ,  
then  $A + P =$

- A) 2
- B) 0
- C) 3
- D)  $\frac{3}{2}$
- E)  $1 + \sqrt{5}$

Double Angle  
Identities.

$$\frac{\csc^2 x + \csc^4 x}{2 + \csc^2 x - \csc^4 x} =$$

- A)  $-\sec 2x$
- B)  $\sec 2x$
- C)  $\csc 2x$
- D)  $-\csc 2x$
- E)  $\cot 2x$

Double Angle  
Identities.

$$(\sin^\circ 15 + \cos^\circ 15)^2 + \frac{2\tan\frac{\pi}{3}}{1-\tan^2\frac{\pi}{3}}$$
 equals to

- A)  $\frac{3-2\sqrt{3}}{2}$
- B)  $\frac{3+\sqrt{3}}{2}$
- C)  $\frac{3+2\sqrt{3}}{2}$
- D)  $-\frac{3+2\sqrt{3}}{2}$
- E)  $\frac{3-\sqrt{3}}{2}$

Double Angle  
Identities.

$$3 - 8 \sin^2(22.5^\circ) \cos^2(22.5^\circ) =$$

A) 2

B) 0

C) 1

D) -1

E) -2

Double Angle  
Identities.

$$\frac{\cot \frac{\pi}{12} - \tan \frac{\pi}{12}}{\cot \frac{\pi}{12} + \tan \frac{\pi}{12}} =$$

A)  $\frac{\sqrt{3}}{2}$

B)  $\frac{1}{2}$

C)  $\frac{\sqrt{3}}{3}$

D)  $\sqrt{3}$

E)  $\frac{\sqrt{3}}{4}$

Double Angle  
Identities.

$$\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} - \frac{\cos \theta - \sin \theta}{\cos \theta + \sin \theta} =$$

A)  $2\tan 2\theta$

B)  $2\cot 2\theta$

C)  $2\sec 2\theta$

D)  $2\csc 2\theta$

E)  $2\sin 2\theta$

Double Angle  
Identities.

$$\tan\left(2\sin^{-1}\frac{2}{\sqrt{13}}\right) =$$

A)  $\frac{12}{5}$

B)  $\frac{4}{3}$

C)  $\frac{5}{12}$

D)  $\frac{4}{5}$

E)  $\frac{3}{5}$

Double Angle  
Identities.

If  $\tan x = 3$ ,  $\sin x < 0$ , then  $\sin 2x + \cos 2x =$

A)  $-\frac{1}{5}$

B)  $\frac{7}{5}$

C)  $\frac{6}{5}$

D)  $-\frac{4}{5}$

E) 1

Double Angle  
Identities.

$$\frac{1}{2} \sin 15^\circ \sin 75^\circ =$$

A)  $\frac{1}{8}$

B)  $-\frac{1}{4}$

C)  $-\frac{\sqrt{3}}{2}$

D)  $\frac{\sqrt{3}}{4}$

E)  $\frac{\sqrt{3}}{8}$

Double Angle  
Identities.

If  $p = \sin 165^\circ \cos 165^\circ$  and  $q = \cos^2 \frac{\pi}{8} - \frac{1}{2}$  then  $p + q =$

A)  $\frac{\sqrt{2}-1}{4}$

B)  $\frac{\sqrt{2}-1}{2}$

C)  $\frac{\sqrt{2}}{4}$

D)  $\frac{\sqrt{2}+1}{2}$

E)  $\frac{1-\sqrt{2}}{2}$

Double Angle  
Identities.

If  $\sin x = -\frac{3}{5}$ ,  $\pi < x < \frac{3\pi}{2}$ , then  $48\cot 2x + 7\sec 2x =$

A) 39

B) -39

C) 11

D) -11

E) 32

Double Angle  
Identities.

If  $\cos 2\theta = \frac{1}{2}$ , where  $\pi < \theta < \frac{3\pi}{2}$ , then  $\cot \theta =$

A)  $\sqrt{3}$

B)  $-\sqrt{2}$

C)  $\frac{\sqrt{3}}{2}$

D)  $\frac{\sqrt{2}}{3}$

E)  $3\sqrt{2}$

Double Angle  
Identities.

The exact value of  $\tan \left[ 2\cos^{-1} \frac{1}{4} \right] =$

A)  $-\frac{\sqrt{15}}{7}$

B)  $\frac{\sqrt{15}}{4}$

C)  $-\frac{\sqrt{13}}{7}$

D)  $\frac{\sqrt{17}}{7}$

E) 34

Double Angle  
Identities.

$\cot \left[ 2\cos^{-1} \left( -\frac{4}{5} \right) \right] =$

A)  $-\frac{7}{24}$

B)  $-\frac{24}{25}$

C)  $-\frac{5}{12}$

D)  $\frac{24}{7}$

E)  $\frac{7}{24}$

Double Angle  
Identities.

Which one of the following equations is an identity?

A)  $\sin^4 x - \cos^4 x = -\cos 2x$

B)  $\sin \frac{x}{2} \cos \frac{x}{2} = 2\sin x$ .

C)  $\tan^2 x - \sec^2 x = 1$

D)  $\sec(x) + \sec(-x) = 0$

E)  $\sin x = \sqrt{1 - \cos^2 x}$

Trigonometric  
Identities.

