CHAPTER 7.2

Addition and Subtraction Formulas

Multiple-Choice Old Exam Questions

46 questions • 5 topics • options A–E • two questions per page

Topic 1 | Writing a sin x + b cos x as a Single Sine or Cosine

Finding k, the phase angle, the amplitude or the period when a combination of sine and cosine is rewritten as one wave • Questions 1–12

Q 1

If the function $y = -3\sin 2x - 3\cos 2x$ is written in the form $y = k\sin(2x + \beta)$, $0 < \beta < 2\pi$, then the values of k and β are

A) $k = 3\sqrt{2},\ \beta = \frac{5\pi}{4}$

B) $k = -6,\ \beta = \frac{5\pi}{4}$

C) $k = 3\sqrt{2},\ \beta = \frac{5\pi}{8}$

D) $k = -6,\ \beta = \frac{3\pi}{4}$

E) $k = 3\sqrt{2},\ \beta = \frac{7\pi}{4}$

Q 2

The expression $2\sin\frac{x}{3} - 2\sqrt{3}\cos\frac{x}{3}$ can be written as

A) $4\sin\left(\frac{x}{3} - \frac{\pi}{3}\right)$

B) $2\sqrt{3}\sin\left(\frac{x}{3} + \frac{\pi}{3}\right)$

C) $4\sin\left(x + \frac{2\pi}{3}\right)$

D) $2\sin\left(\frac{x}{3} + \frac{\pi}{3}\right)$

E) $2\sin\left(\frac{x}{3} - \frac{2\pi}{3}\right)$

Q
3

If the function $y = \frac{1}{2}\sin\left(\frac{1}{2}x\right) - \frac{\sqrt{3}}{2}\cos\left(\frac{1}{2}x\right) + \frac{11}{2}$ is written in the form $y = k\sin(bx + a) + c$, then $k + b + c =$

A) 7

B) 6

C) 5

D) 4

E) 9

Q
4

If the function $y = -\sin 2x - \sqrt{3}\cos 2x$ is written in the form $y = k\sin(2x + a)$, $0 < a < 2\pi$, then the value of a is

A) $\frac{4\pi}{3}$

B) $\frac{2\pi}{3}$

C) $\frac{5\pi}{6}$

D) $\frac{7\pi}{6}$

E) $-\frac{\pi}{3}$

Q
5

If $f(x) = -2\sin\left(\frac{\pi x}{2}\right) + 2\sqrt{3}\cos\left(\frac{\pi x}{2}\right)$, then the phase shift of the graph of $f(x)$ is equal to

A) $-\frac{4}{3}$

B) $-\frac{3}{4}$

C) -3

D) $\frac{3}{4}$

E) $\frac{4}{3}$

Q
6

If the graph of $y = -\sin 2x + \sqrt{3}\cos 2x$ has a period P and amplitude A, then $P \cdot A =$

A) 2π

B) $\frac{\pi}{2}$

C) 2

D) π

E) 3π

Q
7

If $f(x) = \sin \pi\ x + \sqrt{3} \cos \pi\ x$ is written as $f(x) = k \sin(bx + c)$, then the range R and the period P of $f(x)$ are

A) $R = [-2, 2]$ and $P = 2$

B) $R = (-2, 2)$ and $P = \pi$

C) $R = [0, 2]$ and $P = 2$

D) $R = [0, 2]$ and $P = 1$

E) $R = [-2, 2]$ and $P = -2$

Q
8

If the function $y = 3 \sin x + 3\sqrt{3} \cos x$ is written as $y = k \sin(x + \alpha)$, then $k + \alpha =$

A) $6 + \frac{\pi}{3}$

B) $6 + \frac{2\pi}{3}$

C) $3 + \frac{4\pi}{3}$

D) $3\sqrt{3} + \frac{2\pi}{3}$

E) $6\sqrt{3} + \frac{5\pi}{3}$

Q
9

If $\sin 40° + \cos 40° = k \sin \beta$, then

A) $k = \sqrt{2},\ \beta = 85°$

B) $k = \sqrt{2},\ \beta = -45°$

C) $k = 2,\ \beta = 40°$

D) $k = \sqrt{2},\ \beta = 80°$

E) $k = 2,\ \beta = -80°$

Q
10

If $y = \frac{1}{2}\sin x - \frac{\sqrt{3}}{2\sec x}$, then it can be written as:

A) $y = \sin(x - 60°)$

B) $y = 2\sin(x + 60°)$

C) $y = \sin(x - 30°)$

D) $y = \sin(x + 30°)$

E) $y = 2\sin(x - 60°)$

Q
11

If $f(x) = 2\sin\frac{x}{3} - 2\sqrt{3}\cos\frac{x}{3}$ is written in the form $A\sin(Bx + C)$, where $A > 0$, $B > 0$ and $-\frac{\pi}{2} < C < 0$, then the graph of f has:

A) Amplitude 4, phase shift π units to the right.

B) Amplitude 2, phase shift $\frac{\pi}{3}$ units to the right.

C) Amplitude 4, phase shift π units to the left.

D) Amplitude -4, phase shift $\frac{\pi}{3}$ units to the left.

E) Amplitude $2 + 2\sqrt{3}$, phase shift π units to the left.

Q
12

If $\cos 55° - \sin 55° = k\cos\theta$, where $k > 0$ and $0 \leq \theta \leq 180°$, then

A) $k = \sqrt{2},\ \theta = 100°$

B) $k = 2,\ \theta = 100°$

C) $k = \sqrt{2},\ \theta = 80°$

D) $k = \sqrt{2},\ \theta = 70°$

E) $k = \sqrt{2},\ \theta = 120°$

Topic 2 | Range and Extreme Values

Finding the range, minimum value or maximum of functions made of sine and cosine terms • Questions 13–20

Q **13**

The range of $y = \frac{\pi}{\csc x} - \frac{\pi}{\sec x}$ is:

A) $[-\pi\sqrt{2}, \pi\sqrt{2}]$

B) $[-\sqrt{2}, \sqrt{2}]$

C) $[-\pi, \pi]$

D) $\left[-\frac{\sqrt{2}}{\pi}, \frac{\sqrt{2}}{\pi}\right]$

E) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Q **14**

If the range of the function $f(x) = 2 + \cos(3x) + \frac{\sqrt{3}}{\csc(3x)}$ is $[m, n]$, then $m + n =$

A) 8

B) 3

C) 0

D) 4

E) 6

Q
15

If the range of the function $f(x) = 4\sin x + 3\cos x - 1$ is $[m, n]$, then $m + n =$

A) -2

B) -3

C) -6

D) -1

E) -4

Q
16

The minimum value of the function $y = 2 - \sqrt{3}\sin 2x + \cos 2x$ is equal to

A) 0

B) -1

C) 1

D) 2

E) -2

Q
17

If the range of the function $f(x) = 3 + \sin(2x) + \frac{\sqrt{3}}{\sec(2x)}$ is $[m, n]$, then $m + n =$

A) 6

B) 3

C) 10

D) 4

E) 8

Q
18

The range of the function $f(x) = \sin 3x - \frac{3}{4}\cos 3x - 1$ is

A) $\left[-\frac{9}{4}, \frac{3}{4}\right]$

B) $\left[-\frac{9}{2}, \frac{1}{2}\right]$

C) $\left[-\frac{9}{4}, \frac{1}{4}\right]$

D) $\left[-\frac{5}{4}, \frac{3}{4}\right]$

E) $\left[-\frac{7}{4}, \frac{1}{4}\right]$

Q
19

The minimum value of the function $f(x) = -\frac{\sqrt{3}}{2}\sin x - \frac{1}{2}\cos x$ is

A) -1

B) 0

C) $-\frac{\sqrt{3}}{2}$

D) $-\frac{1}{2}$

E) $\frac{-\sqrt{3}-1}{2}$

Q
20

The graph of the function $f(x) = -\sin x - \cos x, -\frac{\pi}{4} \leq x \leq \frac{7\pi}{4}$, is increasing on

A) $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ and $\left[\frac{5\pi}{4}, \frac{7\pi}{4}\right]$

B) $\left[-\frac{\pi}{4}, \frac{3\pi}{4}\right]$

C) $\left[\frac{\pi}{4}, \frac{7\pi}{4}\right]$

D) $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$

E) $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$ and $\left[\frac{5\pi}{4}, \frac{7\pi}{4}\right]$

Topic 3 | Exact Values with Sum and Difference Formulas

Using sine, cosine and tangent of a sum or difference to evaluate special angles and products • Questions 21–30

Q **21**

If $\cos\left(\frac{17\pi}{12}\right) = \frac{\sqrt{a}-\sqrt{b}}{4}$, then $a + b =$

A) 4

B) 5

C) 8

D) 7

E) 6

Q **22**

$$\left(\tan\frac{5\pi}{12}\right)\left(\tan\frac{\pi}{3}\right) =$$

A) $3 + 2\sqrt{3}$

B) $2 + \sqrt{3}$

C) $2 + 3\sqrt{2}$

D) $2\sqrt{3}$

E) $3\sqrt{2}$

Q
23

The value of the expression $\sin 27° \cos 57° - \sin 63° \cos 33°$ is equal to

A) $-\frac{1}{2}$

B) $-\frac{\sqrt{3}}{2}$

C) 0

D) $\frac{1}{2}$

E) $\frac{\sqrt{3}}{2}$

Q
24

$$\tan\left(\frac{11\pi}{12}\right) =$$

A) $\sqrt{3} - 2$

B) $\sqrt{3} - 1$

C) $\frac{\sqrt{3}-1}{2}$

D) $1 - \sqrt{3}$

E) $2 - \sqrt{3}$

Q
25

$$\cos 465° =$$

A) $\frac{\sqrt{2}-\sqrt{6}}{4}$

B) $\frac{\sqrt{6}-\sqrt{2}}{4}$

C) $\frac{\sqrt{2}-\sqrt{6}}{2}$

D) $\frac{\sqrt{3}-\sqrt{6}}{4}$

E) $\frac{\sqrt{6}-\sqrt{2}}{2}$

Q
26

$$\cos\frac{3\pi}{5}\sin\frac{\pi}{10}-\sin\frac{3\pi}{5}\sin\frac{2\pi}{5}=$$

A) -1

B) 1

C) $\frac{3}{5}$

D) $-\frac{3}{5}$

E) 0

Q
27

$$\tan 105° =$$

A) $-2-\sqrt{3}$

B) $\sqrt{3}-2$

C) $\frac{\sqrt{3}-2}{2}$

D) $\frac{1+\sqrt{3}}{4}$

E) $2\sqrt{3}-1$

Q
28

$$\sin 70° \sin 50° - \sin 20° \sin 40° =$$

A) $\frac{1}{2}$

B) $\frac{\sqrt{2}}{2}$

C) $-\frac{\sqrt{3}}{2}$

D) $\frac{\sqrt{3}}{2}$

E) $-\frac{\sqrt{2}}{2}$

Q
29

$$\cos(255°) =$$

A) $\frac{\sqrt{2}-\sqrt{6}}{4}$

B) $\frac{\sqrt{6}-\sqrt{2}}{4}$

C) $\frac{\sqrt{6}+\sqrt{2}}{4}$

D) $\frac{\sqrt{2}-\sqrt{6}}{2}$

E) $\frac{\sqrt{2}+\sqrt{6}}{2}$

Q
30

The exact value of $\frac{1-\cot(70°)\cot(80°)}{\tan(20°)+\cot(80°)}$ is equal to

A) $\sqrt{3}$

B) $2\sqrt{2}$

C) 1

D) $\frac{\sqrt{3}}{3}$

E) $\frac{\sqrt{3}}{2}$

Topic 4 | Inverse Trigonometric Functions and Sum Formulas

Evaluating sines, cosines and tangents of sums that contain inverse trigonometric functions • Questions 31–34

Q **31**

$$\sin\left[\cos^{-1}\frac{1}{2}+\tan^{-1}(-3)\right]=$$

A) $\frac{\sqrt{10}}{20}\left(\sqrt{3}-3\right)$

B) $\frac{\sqrt{10}}{20}\left(\sqrt{3}+3\right)$

C) $\frac{\sqrt{10}}{10}\left(\sqrt{3}-3\right)$

D) $\frac{\sqrt{10}}{10}\left(2\sqrt{3}-1\right)$

E) $\frac{\sqrt{10}}{20}\left(3-\sqrt{3}\right)$

Q **32**

$$\cos\left(\frac{\pi}{4}+\tan^{-1}\frac{3}{4}\right)=$$

A) $\frac{\sqrt{2}}{10}$

B) $\frac{\sqrt{3}}{10}$

C) $\frac{\sqrt{2}}{4}$

D) $\frac{\sqrt{2}}{2}$

E) $\frac{\sqrt{3}}{2}$

Q
33

$$\sin\left(\tan^{-1}\frac{3}{4}+\cos^{-1}\frac{5}{13}\right)=$$

A) $\frac{63}{65}$

B) $-\frac{63}{65}$

C) $\frac{54}{65}$

D) $\frac{33}{65}$

E) $-\frac{33}{65}$

Q
34

$$\sin\left(\tan^{-1}\left(\frac{4}{3}\right)-\cos^{-1}\left(\frac{12}{13}\right)\right)=$$

A) $\frac{33}{65}$

B) $\frac{63}{65}$

C) $\frac{7}{65}$

D) $\frac{9}{13}$

E) $-\frac{33}{65}$

Topic 5 | Sums and Differences from Given Ratios and Quadrants

Using given ratios and quadrants to find sines, cosines or tangents of sums and differences • Questions 35–46

Q **35**

If $\cos(x+y)=1$ and $\cos(x-y)=1$, then $\cos x \cos y =$

A) 1

B) -1

C) 2

D) -2

E) 0

Q **36**

If A is the amplitude and P is the period of the function $y = \cos 3x \cos x - \sin 3x \sin x$, then $\pi A + 2P =$

A) 2π

B) 0

C) 4π

D) π

E) 3π

Q
37

If $\sin\alpha = \frac{4}{5}, -\frac{3\pi}{2} < \alpha < -\pi$, and $\cos\beta = -\frac{\sqrt{5}}{5}, \pi < \beta < \frac{3\pi}{2}$, then $\cos(\alpha + \beta) =$

A) $\frac{3\sqrt{5}}{25}$

B) $-\frac{3\sqrt{5}}{25}$

C) $\frac{11\sqrt{5}}{25}$

D) $-\frac{\sqrt{5}}{25}$

E) $\frac{14\sqrt{5}}{25}$

Q
38

If $\tan\alpha = \frac{3}{2}$ and $\tan\beta = -2$, then $\cot\left(\frac{\pi}{2} - \alpha + \beta\right) =$

A) $-\frac{7}{4}$

B) $-\frac{1}{8}$

C) $-\frac{4}{7}$

D) -8

E) -2

Q
39

If $\cos\alpha = \frac{1}{\sqrt{5}}$, $0 < \alpha < \frac{\pi}{2}$ and $\cos\beta = \frac{1}{\sqrt{10}}$, $\frac{3\pi}{2} < \beta < 2\pi$, then $\tan(\alpha - \beta) =$

A) -1

B) $-\frac{1}{7}$

C) $\frac{1}{7}$

D) $\frac{1}{5}$

E) $-\frac{1}{5}$

Q
40

If $\cos\left(\frac{\pi}{2} - \alpha\right) = \frac{5}{13}$ and $\sec\beta = -\frac{5}{3}$, where α is in quadrant I and β is in quadrant II, then $\cos(\alpha + \beta)$ is equal to

A) $-\frac{56}{65}$

B) $-\frac{33}{65}$

C) $-\frac{16}{65}$

D) $\frac{56}{65}$

E) $\frac{33}{65}$

Q
41

If $\tan\alpha = \frac{3}{2}$ and $\tan\beta = -2$, then $\tan(\alpha - \beta) =$

A) $-\frac{7}{4}$

B) $\frac{7}{2}$

C) $-\frac{1}{2}$

D) $\frac{7}{8}$

E) $\frac{1}{4}$

Q
42

$$\sin\left(\frac{3\pi}{2} + \theta\right) + \cos\left(\frac{3\pi}{2} - \theta\right) =$$

A) $-\sin\theta - \cos\theta$

B) $\cos\theta - \sin\theta$

C) $\sin\theta - \cos\theta$

D) $-2\sin\theta$

E) $\sin\theta + \cos\theta$

Q
43

If s and t are angles in standard position, with $\sin s = \frac{4}{5}, \frac{\pi}{2} < s < \pi$, and $\cos t = -\frac{5}{13}, \pi < t < \frac{3\pi}{2}$, then the terminal side of the angle $s + t$ is in the quadrant(s):

A) I

B) II

C) IV

D) I or II

E) II or III

Q
44

The expression $\frac{1+\tan 100° \tan(-80°)}{\tan 100° - \tan(-80°)}$ is

A) undefined

B) equal to 0

C) equal to -1

D) equal to 1

E) equal to $-\sqrt{3}$

Q
45

If $\cos\alpha = -\frac{4}{5}$, where $\frac{\pi}{2} < \alpha < \pi$, and $\cos\left(\frac{\pi}{2} - \beta\right) = -\frac{12}{13}$, where $\pi < \beta < \frac{3\pi}{2}$, then $\sin(\alpha + \beta)$ is equal to

A) $\frac{33}{65}$

B) $-\frac{7}{65}$

C) $-\frac{63}{65}$

D) $\frac{61}{65}$

E) $-\frac{16}{65}$

Q
46

$$\frac{1 - \tan\frac{13\pi}{9}\tan\frac{2\pi}{9}}{\tan\frac{13\pi}{9} + \tan\frac{2\pi}{9}} =$$

A) $-\frac{\sqrt{3}}{3}$

B) $\cot\frac{11\pi}{9}$

C) $-\cot\frac{11\pi}{9}$

D) $-\tan\frac{11\pi}{9}$

E) $\sqrt{3}$

Answer Key

"Original" shows the question number in the source worksheet.

Q	Original	Ans.	Value
1	1	**A**	$k = 3\sqrt{2},\ \beta = \frac{5\pi}{4}$
2	6	**A**	$4\sin\left(\frac{x}{3} - \frac{\pi}{3}\right)$
3	9	**A**	7
4	10	**A**	$\frac{4\pi}{3}$
5	11	**A**	$-\frac{4}{3}$
6	13	**A**	2π
7	14	**A**	$R = [-2,\ 2]$ and $P = 2$
8	16	**A**	$6 + \frac{\pi}{3}$
9	17	**A**	$k = \sqrt{2},\ \beta = 85°$
10	19	**A**	$y = \sin(x - 60°)$
11	21	**A**	Amplitude 4, phase shift π units to the right.
12	22	**A**	$k = \sqrt{2},\ \theta = 100°$
13	2	**A**	$[-\pi\sqrt{2},\ \pi\sqrt{2}]$
14	3	**D**	4
15	4	**A**	-2
16	5	**A**	0
17	12	**A**	6
18	15	**C**	$\left[-\frac{9}{4}, \frac{1}{4}\right]$
19	20	**A**	-1
20	18	**D**	$\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$
21	7	**C**	8
22	23	**A**	$3 + 2\sqrt{3}$
23	24	**A**	$-\frac{1}{2}$
24	25	**A**	$\sqrt{3} - 2$
25	29	**A**	$\frac{\sqrt{2} - \sqrt{6}}{4}$
26	30	**A**	-1
27	39	**A**	$-2 - \sqrt{3}$
28	40	**A**	$\frac{1}{2}$
29	46	**A**	$\frac{\sqrt{2} - \sqrt{6}}{4}$
30	47	**A**	$\sqrt{3}$
31	26	**A**	$\frac{\sqrt{10}}{20}(\sqrt{3} - 3)$

Q	Original	Ans.	Value
32	27	**A**	$\frac{\sqrt{2}}{10}$
33	31	**A**	$\frac{63}{65}$
34	42	**A**	$\frac{33}{65}$
35	8	**A**	1
36	28	**A**	2π
37	32	**C**	$\frac{11\sqrt{5}}{25}$
38	33	**A**	$-\frac{7}{4}$
39	34	**A**	-1
40	35	**A**	$-\frac{56}{65}$
41	37	**A**	$-\frac{7}{4}$
42	38	**A**	$-\sin\theta - \cos\theta$
43	41	**A**	I
44	43	**A**	undefined
45	44	**A**	$\frac{33}{65}$
46	45	**A**	$-\frac{\sqrt{3}}{3}$