7.2: (Addition and Subtraction Formulas)

If the function $y = -3\sin 2x - 3\cos 2x$ is written in the form $y = k\sin(2x + \beta),\ 0 < \beta < 2\pi,$ then the values of k and β are

(a) $k = 3\sqrt{2}, \beta = \frac{5\pi}{4}$

(b) $k = -6, \beta = \frac{5\pi}{4}$

(c) $k = 3\sqrt{2}, \beta = \frac{5\pi}{8}$

(d) $k = -6, \beta = \frac{3\pi}{4}$

(e) $k = 3\sqrt{2}, \beta = \frac{7\pi}{4}$

If $\cos(x + y) = 1$, and $\cos(x - y) = 1$, then $\cos x \cos y =$

(A) 1

(B) -1

(C) 2

(D) -2

(E) 0

The range of $y = \frac{\pi}{\csc x} - \frac{\pi}{\sec x}$ is:

A) $[-\pi\sqrt{2}, \pi\sqrt{2}]$

B) $[-\sqrt{2}, \sqrt{2}]$

C) $[-\pi, \pi]$

D) $\left[-\frac{\sqrt{2}}{\pi}, \frac{\sqrt{2}}{\pi}\right]$

E) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

If the range of the function $f(x) = 2 + \cos(3x) + \frac{\sqrt{3}}{\csc(3x)}$ is $[m, n]$. then

$m + n =$

A) 8

B) 3

C) 0

D) 4

E) 6

If the range of the function $f(x) = 4\sin x + 3\cos x - 1$ is $[m, n]$, then $m + n =$

(a) -2

(b) -3

(c) -6

(d) -1

(e) -4

The minimum value of the function $y = 2 - \sqrt{3}\sin 2x + \cos 2x$ is equal to

A) 0

B) -1

C) 1

D) 2

E) - 2

The expression $2\sin\frac{x}{3} - 2\sqrt{3}\cos\frac{x}{3}$ can be written as

A) $4\sin\left(\frac{x}{3} - \frac{\pi}{3}\right)$

B) $2\sqrt{3}\sin\left(\frac{x}{3} + \frac{\pi}{3}\right)$

C) $4\sin\left(x + \frac{2\pi}{3}\right)$

D) $2\sin\left(\frac{x}{3} + \frac{\pi}{3}\right)$

E) $2\sin\left(\frac{x}{3} - \frac{2\pi}{3}\right)$

If $\cos\left(\frac{17\pi}{12}\right) = \frac{\sqrt{a}-\sqrt{b}}{4}$, then $a + b =$

A) 4

B) 5

C) 8

D) 7

E) 6

If $\cos(x+y)=1$, and $\cos(x-y)=1$, then $\cos x \cos y =$

(a) 1

(b) -1

(c) 2

(d) -2

(e) 0

If the function $y=\frac{1}{2}\sin\left(\frac{1}{2}x\right)-\frac{\sqrt{3}}{2}\cos\left(\frac{1}{2}x\right)+\frac{11}{2}$ is written in the form $y=k\sin(bx+a)+c$, then $k+b+c=$

A) 7

B) 6

C) 5

D) 4

E) 9

If the function $y = -\sin 2x - \sqrt{3}\cos 2x$ is written in the form $y = k\sin(2x + a)$,$0 < a < 2\pi$, then the value of a is

A) $\frac{4\pi}{3}$

B) $\frac{2\pi}{3}$

C) $\frac{5\pi}{6}$

D) $\frac{7\pi}{6}$

E) $-\frac{\pi}{3}$

If $f(x) = -2\sin\left(\frac{\pi x}{2}\right) + 2\sqrt{3}\cos\left(\frac{\pi x}{2}\right)$, then the phase shift of the graph of $f(x)$ is equal to

A) $-\frac{4}{3}$

B) $-\frac{3}{4}$

C) -3

D) $\frac{3}{4}$

E) $\frac{4}{3}$

If the range of the function $f(x) = 3 + \sin(2x) + \frac{\sqrt{3}}{\sec(2x)}$ is $[m, n]$ then

$m + n =$

A) 6

B) 3

C) 10

D) 4

E) 8

If the graph of $y = -\sin 2x + \sqrt{3}\cos 2x$ has a period P and amplitude A,

then $\mathrm{P} \cdot \mathrm{A} =$

A) 2π

B) $\frac{\pi}{2}$

C) 2

D) π

E) 3π

If $f(x) = \sin \pi x + \sqrt{3}\cos \pi x$ is written as $f(x) = k\sin(bx + c)$, then the range R and the period P of $f(x)$ are

A) $R = [-2,2]$ and $P = 2$

B) $R = (-2,2)$ and $P = \pi$

C) $R = [0,2]$ and $P = 2$

D) $A = [0,2]$ and $P = 1$

E) $A = [-2,2]$ and $P = -2$

The range of the function $f(x) = \sin 3x - \frac{3}{4}\cos 3x - 1$ is

A) $\left[\frac{-9}{4}, \frac{3}{4}\right]$

B) $\left[\frac{-9}{2}, \frac{1}{2}\right]$

C) $\left[\frac{-9}{4}, \frac{1}{4}\right]$

D) $\left[\frac{-5}{4}, \frac{3}{4}\right]$

E) $\left[\frac{-7}{4}, \frac{1}{4}\right]$

If the function $y = 3\sin x + 3\sqrt{3}\cos x$ is written as $y = k\sin(x+\alpha)$, then $k + \alpha =$

A) $6 + \frac{\pi}{3}$

B) $6 + \frac{2\pi}{3}$

C) $3 + \frac{4\pi}{3}$

D) $3\sqrt{3} + \frac{2\pi}{3}$

E) $6\sqrt{3} + \frac{5\pi}{3}$

If $\sin 40° + \cos 40° = k\sin\beta$, then

A) $k = \sqrt{2}, \beta = 85°$

B) $k = \sqrt{2}, \beta = -45°$

C) $k = 2, \beta = 40°$

D) $k = \sqrt{2}, \beta = 80°$

E) $k = 2,\ \beta = -80°$

The graph of the function $f(x) = -\sin x - \cos x, -\frac{\pi}{4} \leq x \leq \frac{7\pi}{4}$, is increasing on

A) $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ and $\left[\frac{5\pi}{4}, \frac{7\pi}{4}\right]$

B) $\left[-\frac{\pi}{4}, \frac{3\pi}{4}\right]$

C) $\left[\frac{\pi}{4}, \frac{7\pi}{4}\right]$

D) $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$

E) $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$ and $\left[\frac{5\pi}{4}, \frac{7\pi}{4}\right]$

If $y = \frac{1}{2}\sin x - \frac{\sqrt{3}}{2\sec x}$, then it can be written as:

A) $y = \sin(x - 60°)$

B) $y = 2\sin(x + 60°)$

C) $y = \sin(x - 30°)$

D) $y = \sin(x + 30°)$

E) $y = 2\sin(x - 60°)$

The minimum value of the function $f(x) = -\frac{\sqrt{3}}{2}\sin x - \frac{1}{2}\cos x$ is

A) -1

B) 0

C) $-\frac{\sqrt{3}}{2}$

D) $-\frac{1}{2}$

E) $\frac{-\sqrt{3}-1}{2}$

If $f(x) = 2\sin\frac{x}{3} - 2\sqrt{3}\cos\frac{x}{3}$ is written in the form $A\sin(Bx + C)$ where $A > 0, B > 0$ and $-\frac{\pi}{2} < C < 0$, then the graph of f has:

A) Amplitude 4, phase shift π units to the right.

B) Amplitude 2, phase shift $\frac{\pi}{3}$ units to the right.

C) Amplitude 4, phase shift π units to the left.

D) Amplitude - 4, phase shift $\frac{\pi}{3}$ units to the left.

E) Amplitude $2 + 2\sqrt{3}$, phase shift π units to the left.

If $\cos 55° - \sin 55° = k\cos\theta$, where $k > 0$ and $0 \le \theta \le 180°$, then

(a) $k = \sqrt{2},\ \theta = 100°$

(b) $k = 2,\ \theta = 100°$

(c) $k = \sqrt{2},\ \theta = 80°$

(d) $k = \sqrt{2},\ \theta = 70°$

(e) $k = \sqrt{2},\ \theta = 120°$

$$\left(\tan\frac{5\pi}{12}\right)\left(\tan\frac{\pi}{3}\right)$$

A) $3 + 2\sqrt{3}$

B) $2 + \sqrt{3}$

C) $2 + 3\sqrt{2}$

D) $2\sqrt{3}$

E) $3\sqrt{2}$

The value of the expression $\sin 27^{\circ}\cos 57^{\circ} - \sin 63^{\circ}\cos 33^{\circ}$ is equal to

A) $-\frac{1}{2}$

B) $-\frac{\sqrt{3}}{2}$

c) 0

D) $\frac{1}{2}$

E) $\frac{\sqrt{3}}{2}$

$\tan\left(\frac{11\pi}{12}\right) =$

A) $\sqrt{3} - 2$

B) $\sqrt{3} - 1$

C) $\frac{\sqrt{3}-1}{2}$

D) $1 - \sqrt{3}$

E) $2 - \sqrt{3}$

$\sin\left[\cos^{-1}\frac{1}{2} + \tan^{-1}(-3)\right] =$

A) $\frac{\sqrt{10}}{20}(\sqrt{3} - 3)$
B) $\frac{\sqrt{10}}{20}(\sqrt{3} + 3)$
C) $\frac{\sqrt{10}}{10}(\sqrt{3} - 3)$
D) $\frac{\sqrt{10}}{10}(2\sqrt{3} - 1)$
E) $\frac{\sqrt{10}}{20}(3 - \sqrt{3})$

$\cos\left(\frac{\pi}{4} + \tan^{-1}\frac{3}{4}\right) =$

A) $\frac{\sqrt{2}}{10}$

B) $\frac{\sqrt{3}}{10}$

C) $\frac{\sqrt{2}}{4}$

D) $\frac{\sqrt{2}}{2}$

E) $\frac{\sqrt{3}}{2}$

If A is the amplitude and P is the period of the function $y = \cos 3x \cos x - \sin 3x \sin x$, then $\pi A + 2P =$

A) 2π

B) 0

C) 4π

D) π

E) 3π

$\cos 465^\circ =$

A) $\frac{\sqrt{2}-\sqrt{6}}{4}$

B) $\frac{\sqrt{6}-\sqrt{2}}{4}$

C) $\frac{\sqrt{2}-\sqrt{6}}{2}$

D) $\frac{\sqrt{3}-\sqrt{6}}{4}$

E) $\frac{\sqrt{6}-\sqrt{2}}{2}$

$$\cos\frac{3\pi}{5}\sin\frac{\pi}{10} - \sin\frac{3\pi}{5}\sin\frac{2\pi}{5} =$$

A) -1

B) 1

C) $\frac{3}{5}$

D) $-\frac{3}{5}$

E) 0

$$\sin\left(\tan^{-1}\frac{3}{4} + \cos^{-1}\frac{5}{13}\right) =$$

A) $\frac{63}{65}$

B) $-\frac{63}{65}$

C) $\frac{54}{65}$

D) $\frac{33}{65}$

E) $-\frac{33}{65}$

If $\sin\alpha = \frac{4}{5}, -\frac{3\pi}{2} < \alpha < -\pi$, and $\cos\beta = -\frac{\sqrt{5}}{5}, \pi < \beta < \frac{3\pi}{2}$, then $\cos(\alpha + \beta) =$

A) $\frac{3\sqrt{5}}{25}$

B) $-\frac{3\sqrt{5}}{25}$

C) $\frac{11\sqrt{5}}{25}$

D) $-\frac{\sqrt{5}}{25}$

E) $\frac{14\sqrt{5}}{25}$

If $\tan\alpha = \frac{3}{2}$ and $\tan\beta = -2$, then $\cot\left(\frac{\pi}{2} - \alpha + \beta\right) =$

A) $-\frac{7}{4}$

B) $-\frac{1}{8}$

C) $-\frac{4}{7}$

D) -8

E) -2

If $\cos\alpha = \frac{1}{\sqrt{5}}, 0 < \alpha < \frac{\pi}{2}$ and $\cos\beta = \frac{1}{\sqrt{10}}, \frac{3\pi}{2} < \beta < 2\pi$, then $\tan(\alpha - \beta) =$

A) -1

B) $-\frac{1}{7}$

C) $\frac{1}{7}$

D) $\frac{1}{5}$

E) $-\frac{1}{5}$

If $\cos\left(\frac{\pi}{2} - \alpha\right) = \frac{5}{13}$ and $\sec\beta = -\frac{5}{3}$, where α is in quadrant I and β is in quadrant II, then $\cos(\alpha + \beta)$ is equal to

A) $-\frac{56}{65}$

B) $-\frac{33}{65}$

C) $-\frac{16}{65}$

D) $\frac{56}{65}$

E) $\frac{33}{65}$

If $\sin^{-1} x - \sin^{-1}\left(-\frac{3}{5}\right) = \tan^{-1}(-3)$, then $x =$

A) $-\frac{3\sqrt{10}}{10}$

B) $-\frac{9\sqrt{10}}{10}$

C) $\frac{3\sqrt{10}}{10}$

D) $\frac{9\sqrt{10}}{10}$

E) $3\sqrt{10}$

If $\tan\alpha = \frac{3}{2}$ and $\tan\beta = -2$, then $\tan(\alpha - \beta) =$

A) $-\frac{7}{4}$

B) $\frac{7}{2}$

C) $-\frac{1}{2}$

D) $\frac{7}{8}$

E) $\frac{1}{4}$

$\sin\left(\frac{3\pi}{2}+\theta\right)+\cos\left(\frac{3\pi}{2}-\theta\right)=$

A) $-\sin\theta-\cos\theta$

B) $\cos\theta-\sin\theta$

C) $\sin\theta-\cos\theta$

D) $-2\sin\theta$

E) $\sin\theta+\cos\theta$

$\tan 105^\circ=$

A) $-2-\sqrt{3}$

B) $\sqrt{3}-2$

C) $\frac{\sqrt{3}-2}{2}$

D) $\frac{1+\sqrt{3}}{4}$

E) $2\sqrt{3}-1$

$\sin 70° \sin 50° - \sin 20° \sin 40° =$

A) $\frac{1}{2}$

B) $\frac{\sqrt{2}}{2}$

C) $-\frac{\sqrt{3}}{2}$

D) $\frac{\sqrt{3}}{2}$

E) $-\frac{\sqrt{2}}{2}$

If s and t are angles in standard position, with $\sin s = \frac{4}{5}, \frac{\pi}{2} < s < \pi$, and $\cos t = -\frac{5}{13}, \pi < t < \frac{3\pi}{2}$, then the terminal side of the angle $s + t$ is in the quadrant(s):

A) I

B) II

C) IV

D) I or II

E) II or III

$$\sin\left(\tan^{-1}\left(\frac{4}{3}\right)-\cos^{-1}\left(\frac{12}{13}\right)\right)=$$

A) $\frac{33}{65}$

B) $\frac{63}{65}$

C) $\frac{7}{65}$

D) $\frac{9}{13}$

E) $-\frac{33}{65}$

The expression $\frac{1+\tan 100°\tan(-80°)}{\tan 100°-\tan(-80°)}$ is

A) undefined

B) equal to 0

C) equal to -1

D) equal to 1

E) equal to $-\sqrt{3}$

If $\cos\alpha = -\frac{4}{5}$, where $\frac{\pi}{2} < \alpha < \pi$ and $\cos\left(\frac{\pi}{2} - \beta\right) = -\frac{12}{13}$, where $\pi < \beta < \frac{3\pi}{2}$, then $\sin(\alpha + \beta)$ is equal to

A) $\frac{33}{65}$

B) $-\frac{7}{65}$

C) $-\frac{63}{65}$

D) $\frac{61}{65}$

E) $-\frac{16}{65}$

$$\frac{1 - \tan\frac{13\pi}{9}\tan\frac{2\pi}{9}}{\tan\frac{13\pi}{9} + \tan\frac{2\pi}{9}} =$$

A) $-\frac{\sqrt{3}}{3}$

B) $\cot\frac{11\pi}{9}$

C) $-\cot\frac{11\pi}{9}$

D) $-\tan\frac{11\pi}{9}$

E) $\sqrt{3}$

$\cos(255°) =$

(a) $\frac{\sqrt{2}-\sqrt{6}}{4}$

(b) $\frac{\sqrt{6}-\sqrt{2}}{4}$

(c) $\frac{\sqrt{6}+\sqrt{2}}{4}$

(d) $\frac{\sqrt{2}-\sqrt{6}}{2}$

(e) $\frac{\sqrt{2}+\sqrt{6}}{2}$

The exact value of $\frac{1-\cot(70°)\cot(80°)}{\tan(20°)+\cot(80°)}$ is equal to

(a) $\sqrt{3}$

(b) $2\sqrt{2}$

(c) 1

(d) $\frac{\sqrt{3}}{3}$

(e) $\frac{\sqrt{3}}{2}$