CHAPTER 7.1

Trigonometric Identities

Multiple-Choice Old Exam Questions

42 questions • 4 topics • options A–E • two questions per page

Topic 1 | Simplifying Single Trigonometric Expressions

Simplifying products, powers and quotients with the Pythagorean, reciprocal and even-odd identities • Questions 1–14

Q **1**

If $A = 3\sin^2\alpha + 3\cos^2\alpha$ and $B = 4\cot^2\alpha - 4\csc^2\alpha$, then $A + B =$

A) -1

B) 0

C) 7

D) 1

E) -7

Q **2**

The expression $\frac{\tan^2 x}{1+\sec x}$ is identical to

A) $\frac{1-\cos x}{\cos x}$

B) $\frac{1-\cos x}{\sin x}$

C) $\frac{1-\sin x}{\sin x}$

D) $\frac{1+\cos x}{\cos x}$

E) $\frac{1+\sin x}{\sin x}$

Q
3

$\frac{\cot\theta - \tan\theta}{\sin\theta\cos\theta}$ is identical to:

A) $\csc^2\theta - \sec^2\theta$

B) $\sec^2\theta - \csc^2\theta$

C) $1 - \sec^2\theta$

D) $\sec\theta + \csc\theta$

E) $1 - \csc^2\theta$

Q
4

$$\frac{1+\cot^2\theta}{1-\csc^2\theta} = \boxtimes$$

A) $-\sec^2\theta$

B) $\sec^2\theta - 2$

C) $-\csc^2\theta$

D) $\csc^2\theta - 2$

E) $-\csc^2\theta - \sec^2\theta$

Q
5

$$(\sin^2 x)(1+\cot x)+(\cos^2 x)(1-\tan x)+\cot^2 x = \square$$

A) $\csc^2 x$

B) $\sec^2 x$

C) $\tan^2 x$

D) $\cot^2 x$

E) $\cos^2 x$

Q
6

$$\tan^2(-x)-\sin^2(-x)-\cos^2(x) = \square$$

A) $-1+\tan^2 x$

B) $\sec^2 x$

C) $1+\cot^2 x$

D) $\csc^2 x$

E) $\sin^2 x-\cos^2 x$

Q
7

$\frac{\tan x - \cot x}{\tan x + \cot x}$ is identical to

A) $1 - 2\cos^2 x$

B) $1 - \tan^2 x$

C) $1 + \sec^2 x$

D) $1 + 2\sin^2 x$

E) $1 + 2\cos^2 x$

Q
8

$$\sin^3\theta + \cos^3\theta + \sin\theta\cos^2\theta + \sin^2\theta\cos\theta = \boxtimes$$

A) $\sin\theta + \cos\theta$

B) $\sin\theta - \cos\theta$

C) $\cos\theta - \sin\theta$

D) $2(\sin\theta + \cos\theta)$

E) $2\sin\theta\cos\theta$

Q
9

$$(\sin\theta + \csc\theta)^2 + (\cos\theta + \sec\theta)^2 - \tan^2\theta - \cot^2\theta = \square$$

A) 7

B) 3

C) 5

D) 0

E) 1

Q
10

$$\frac{2\tan x \cos^2 x - \tan x}{1 - \tan^2 x} = \square$$

A) $\sin x \cos x$

B) $\cos^2 x$

C) $-\cot x \sin^2 x$

D) $-\cot x \cos^2 x$

E) $-\sec x \csc^2 x$

Q
11

$$\tan^2 \frac{25\pi}{3} - \sec^2 60° + 1 = ?$$

A) 0

B) $\frac{3}{2}$

C) $\frac{7}{4}$

D) $\frac{3}{4}$

E) 2

Q
12

$$\tan^2 x \,(1 + \cot^2 x) = ?$$

A) $\frac{1}{1-\sin^2 x}$

B) $\frac{1}{1-\cos^2 x}$

C) $\csc^2 x$

D) $-\sec^2 x$

E) $-\csc^2 x$

Q
13

When simplified, the expression $(2\sin x + \cos x)^2 + (2\cos x - \sin x)^2 - 5$ is equal to

A) 0

B) -5

C) 1

D) $8\sin x \cos x$

E) $3\sin^2 x + 5\cos^2 x - 5$

Q
14

The expression $(\sin\theta - \cos\theta)(\csc\theta + \sec\theta)$ simplifies to

A) $\tan\theta - \cot\theta$

B) $\sec\theta - \tan\theta$

C) $\csc\theta \cot\theta$

D) 1

E) 0

Topic 2 | Fractions, Conjugates and Squares

Combining or simplifying expressions that contain fractions, conjugates and squared binomials • Questions 15–28

Q 15

$\frac{\cos x - \cot x}{1 - \sin x}$ is identical to

A) $\tan x$

B) $\cot x$

C) $-\tan x$

D) 0

E) $-\cot x$

Q 16

$$\frac{\cos\theta}{1+\sin\theta} + \frac{1+\sin\theta}{\cos\theta} = \boxtimes$$

A) $2\sec\theta$

B) $2\csc\theta$

C) $2\cos\theta$

D) $2\sin\theta$

E) $2\cot\theta$

Q
17

The expression $\frac{\cos x}{1-\sin x} - \tan x$ simplifies to

A) $\sec x$

B) $-\sec x$

C) $\csc x$

D) $-\csc x$

E) $2\tan x$

Q
18

The expression $\frac{1}{1+\csc x} - \frac{1}{1+\csc(-x)}$ simplifies to

A) $2\sin x \sec^2 x$

B) 0

C) $-2\sin x \sec^2 x$

D) $-2\tan^2 x$

E) 2

Q
19

$$\tan\theta + \frac{1}{\sec\theta + \tan\theta} = \boxtimes$$

A) $\sec\theta$

B) $\cos\theta$

C) $1 + \sin\theta$

D) 1

E) $\sin\theta$

Q
20

$$(\cot x - \csc x)^2 = \boxtimes$$

A) $\frac{1-\cos x}{1+\cos x}$

B) $\frac{1+\cos x}{1-\cos x}$

C) $\frac{\cos x-\sin x}{1+\cos x}$

D) $\frac{\sin x}{1-\cos x}$

E) $\frac{\cos x}{1+\cos x}$

Q
21

$$\sec^2 x - 2\sec x \tan x + \tan^2 x = \boxtimes$$

A) $\frac{1-\sin x}{1+\sin x}$

B) $\frac{1+\sin x}{1-\sin x}$

C) $\frac{1+\tan x}{1-\tan x}$

D) $\frac{1-\cos x}{1+\cos x}$

E) $\frac{1+\cos x}{1-\cos x}$

Q
22

$$2\csc^2 x - 2\csc x \cot x - 1 = \boxtimes$$

A) $\frac{1-\cos x}{1+\cos x}$

B) $\frac{1+\cos x}{1-\cos x}$

C) $\frac{1-\sin x}{1+\cos x}$

D) $\frac{1+\sin x}{1-\cos x}$

E) $\frac{\cos x-1}{\sin x+1}$

Q
23

$$(\csc^2 x)(1+\cos x)^2 = \boxtimes$$

A) $\frac{\sec x \csc x + 1}{\sec x \csc x - 1}$

B) $\frac{\sec x + \csc x}{\sec x - \csc x}$

C) $\frac{\sec x + 1}{\sec x - 1}$

D) $\frac{1}{\sec x - 1}$

E) $\frac{\sec x + 1}{\sec x}$

Q
24

$$\frac{\cot^2 \theta}{1+\csc \theta} = \boxtimes$$

A) $\cos\theta - \sin\theta$

B) $\frac{1+\cos\theta}{\sin\theta}$

C) $\frac{1+\sin\theta}{\sin\theta}$

D) $\frac{1-\sin\theta}{\sin\theta}$

E) $\frac{1-\cos\theta}{\cos\theta}$

Q
25

$$\frac{\sin x\,(\tan x + 1) - 2\tan x \cos x}{\sin x - \cos x} = \boxtimes$$

A) $-\tan x$

B) $\tan x$

C) $\sec x$

D) $-\sec x$

E) $\sin x$

Q
26

$$\frac{1}{1+\cos x} - \frac{1}{1-\cos x} = \boxtimes$$

A) $-2\cot x \csc x$

B) $2\tan x \csc x$

C) $2\tan x \sec x$

D) $-2\cot x \sec x$

E) -2

Q
27

$\frac{\cot x+\csc x}{\sin x+\tan x}$ simplifies to:

A) $\csc x \cot x$

B) $\sin x \tan x$

C) $\sin x \cos x$

D) $\sec^2 x \tan x$

E) $\sin x \sec^2 x$

Q
28

$$\frac{2\sec\theta\csc\theta - 2\tan\theta\csc\theta}{(\sec\theta - \tan\theta)^2 + 1} = \boxtimes$$

A) $\cot\theta$

B) $\tan\theta$

C) $2\sec\theta$

D) $2\csc\theta$

E) $2\sec\theta\tan\theta$

Topic 3 | Constants, Logarithms and Expressions in Terms of Another Function

Finding constants in an identity, using logarithms, and writing one ratio in terms of another • Questions 29–38

Q **29**

If $A = \ln(\sec^2 x) - \ln(\tan^2 x)$, then $e^A - 1 =$

A) $\cot^2 x$

B) $\sec^2 x$

C) $\cos^2 x$

D) 0

E) 1

Q **30**

$$\ln\left(e^{\sin^2 x} e^{\cos^2 x}\right) =$$

A) 1

B) 0

C) $e^{\sec x}$

D) $e^{\csc x}$

E) $2\cos x + 2\sin x$

Q
31

If $\frac{(\tan x+\cot x)^2}{\sin^2 x-\sin^4 x} = \sec^m x \csc^n x$, then $m+n =$☒

A) 8

B) 6

C) 4

D) 10

E) 12

Q
32

If $\pi < \theta < \frac{3\pi}{2}$, then $\cos\theta =$☒

A) $-\frac{\cot\theta\sqrt{1+\cot^2\theta}}{1+\cot^2\theta}$

B) $\frac{\cot\theta\sqrt{1+\cot^2\theta}}{1+\cot^2\theta}$

C) $-\sqrt{1+\cot^2\theta}$

D) $\frac{\sqrt{1+\cot^2\theta}}{1+\cot^2\theta}$

E) $-\frac{\sqrt{1+\cot^2\theta}}{1+\cot^2\theta}$

Q
33

If $\frac{\sec x+\csc x}{\tan x+\cot x} = a\sin x + b\cos x$, then $a + b =$

A) 2

B) 1

C) 3

D) 4

E) 0

Q
34

If $\csc\theta = \frac{x+1}{x}$, where $x > 0$ and θ is an acute angle, then $\cot\theta =$

A) $\frac{\sqrt{1+2x}}{x}$

B) $\frac{\sqrt{2x-1}}{x}$

C) $\frac{\sqrt{x^2+2x}}{x}$

D) $\frac{\sqrt{2x^2+2x+1}}{x}$

E) $\frac{1}{x}$

Q
35

If $(\tan x \sin x)^2 = A\tan^2 x + B\sin^2 x$ is an identity, then $A + B =$

A) 0

B) -2

C) 2

D) 1

E) -1

Q
36

The expression $\frac{\sin\theta}{1-\cot\theta} + \frac{\cos\theta}{1-\tan\theta}$ simplifies to:

A) $\sin\theta + \cos\theta$

B) $\sin\theta - \cos\theta$

C) $\frac{1}{\sin\theta - \cos\theta}$

D) $\cos\theta - \sin\theta$

E) $\frac{1}{\cos\theta - \sin\theta}$

Q
37

If $\sin\beta + \cos\beta = \frac{2}{3}$, then $\sin^3\beta + \cos^3\beta =$

A) $\frac{13}{12}$

B) $\frac{8}{27}$

C) $\frac{23}{12}$

D) $\frac{13}{27}$

E) $\frac{23}{27}$

Q
38

If x is in the third quadrant, then which one of the following is TRUE?

A) $\csc x = +\sqrt{1 + \cot^2 x}$

B) $\csc x = +\sqrt{1 - \cot^2 x}$

C) $\sin x = \frac{+\sqrt{\sec^2 x - 1}}{\sec x}$

D) $\csc x = -\sqrt{1 - \cot^2 x}$

E) $\sin x = \frac{-\sqrt{\sec^2 x - 1}}{\sec x}$

Topic 4 | True and False Statements and Further Identities

Deciding which statement about identities is true or false, and proving a three-term identity • Questions 39–42

Q **39**

Which one of the following statements is TRUE?

A) $\tan^2 x = \sin^2 x \tan^2 x + \sin^2 x$

B) $\tan^2 x = 1 + \sec^2 x$

C) $\tan^2 x = (\sec x + 1)^2$

D) $\tan^2 x = 1 - \cot^2 x$

E) $\tan^2 x = \frac{\sin^2 x}{\sin^2 x - 1}$

Q **40**

Which one of the following statements is TRUE?

A) $\sin x = \sqrt{1 - \cos^2 x}$, if $0 \leq x \leq \pi$.

B) $\csc x = \sqrt{1 + \cot^2 x}$

C) $\sqrt{\cos^2 x} = \cos x$ is an identity.

D) $\sin^3 x = \sin x\,(1 + \cos^2 x)$

E) $\cos^4 x + 1 = (\cos^2 x - 1)(\cos^2 x + 1)$

Q
41

$$\frac{\sin^3\theta}{1-\cos\theta}-\frac{\cos^3\theta}{1-\sin\theta}=\boxtimes$$

A) $\sin\theta-\cos\theta$

B) $\sin\theta\cos\theta$

C) $\sin\theta+\cos\theta$

D) $2\sin\theta$

E) $2\cos\theta$

Q
42

Which one of the following statements is FALSE?

A) $\sec\left(\frac{\pi}{2}+x\right)=\csc x$

B) $\tan\left(x-\frac{\pi}{2}\right)=-\cot x$

C) $\sin\left(\frac{\pi}{2}+x\right)=\cos x$

D) $\sin\left(\frac{\pi}{2}-x\right)=\cos x$

E) $\cos\left(\frac{\pi}{2}-x\right)=\sin(\pi-x)$

Answer Key

"Original" shows the question number in the source worksheet.

Q	Original	Ans.	Value
1	7	**A**	-1
2	8	**A**	$\frac{1-\cos x}{\cos x}$
3	10	**A**	$\csc^2\theta - \sec^2\theta$
4	12	**A**	$-\sec^2\theta$
5	16	**A**	$\csc^2 x$
6	17	**A**	$-1+\tan^2 x$
7	19	**A**	$1-2\cos^2 x$
8	21	**A**	$\sin\theta+\cos\theta$
9	26	**A**	7
10	27	**A**	$\sin x\cos x$
11	28	**A**	0
12	35	**A**	$\frac{1}{1-\sin^2 x}$
13	36	**A**	0
14	39	**A**	$\tan\theta-\cot\theta$
15	1	**E**	$-\cot x$
16	2	**A**	$2\sec\theta$
17	5	**A**	$\sec x$
18	13	**A**	$2\sin x\sec^2 x$
19	15	**A**	$\sec\theta$
20	18	**A**	$\frac{1-\cos x}{1+\cos x}$
21	22	**A**	$\frac{1-\sin x}{1+\sin x}$
22	25	**A**	$\frac{1-\cos x}{1+\cos x}$
23	29	**C**	$\frac{\sec x+1}{\sec x-1}$
24	30	**D**	$\frac{1-\sin\theta}{\sin\theta}$
25	31	**B**	$\tan x$
26	32	**A**	$-2\cot x\csc x$
27	33	**A**	$\csc x\cot x$
28	34	**A**	$\cot\theta$
29	4	**A**	$\cot^2 x$
30	6	**A**	1
31	9	**A**	8
32	11	**A**	$-\frac{\cot\theta\sqrt{1+\cot^2\theta}}{1+\cot^2\theta}$
33	14	**A**	2

Q	Original	Ans.	Value
34	23	**A**	$\frac{\sqrt{1+2x}}{x}$
35	24	**A**	0
36	37	**A**	$\sin\theta + \cos\theta$
37	37	**E**	$\frac{23}{27}$
38	38	**C**	$\sin x = \frac{+\sqrt{\sec^2 x - 1}}{\sec x}$
39	3	**A**	$\tan^2 x = \sin^2 x \tan^2 x + \sin^2 x$
40	20	**A**	$\sin x = \sqrt{1 - \cos^2 x}$, if $0 \leq x \leq \pi$.
41	40	**A**	$\sin\theta - \cos\theta$
42	41	**A**	$\sec\left(\frac{\pi}{2} + x\right) = \csc x$