CHAPTER 6.4

More Trigonometric Graphs

Multiple-Choice Old Exam Questions

57 questions • 6 topics • options A–E • two questions per page

Topic 1 | Tangent and Cotangent: Asymptotes, Zeros and Sign

Counting vertical asymptotes and x-intercepts, and finding where tangent and cotangent graphs are above or below the x-axis • Questions 1–14

Q **1**

The number of all vertical asymptotes of the function $y = 2\tan\left(3\pi\, x + \frac{\pi}{4}\right)$, $0 \leq x \leq \frac{5}{12}$, is

A) 2

B) 5

C) 4

D) 1

E) 3

Q **2**

The graph of the function $y = 2\tan\left(2x - \frac{\pi}{2}\right)$, $0 < x < \pi$, is below the x-axis on

A) $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right)$

B) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right) \cup \left(\frac{3\pi}{4}, \pi\right)$

C) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$

D) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

E) $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{3\pi}{4}, \pi\right)$

Q
3

The graph of the function $f(x) = -\tan\left(\frac{\pi}{4}x - \frac{\pi}{2}\right)$, $0 < x < 8$, is completely above the x-axis on

A) $(0, 2) \cup (4, 6)$

B) $(1, 3) \cup (5, 7)$

C) $(0, 3)$

D) $(0, 4)$

E) $(2, 4) \cup (6, 8)$

Q
4

The number of the x-intercepts of the function $f(x) = -3\tan\left(2x - \frac{\pi}{4}\right)$ over the interval $[-\pi, 2\pi]$ is:

A) 6

B) 5

C) 4

D) 3

E) 2

Q
5

The number of the x-intercepts of the graph of $f(x) = 2\tan\left(3x - \frac{\pi}{4}\right)$, where $-\frac{\pi}{4} \leq x \leq \frac{3\pi}{4}$, is

A) 4

B) 3

C) 1

D) 5

E) 2

Q
6

The number of vertical asymptotes of the graph of the function $y = 2\cot\left(\pi x - \frac{\pi}{3}\right)$, $-1 \leq x \leq 1$, is equal to

A) 5

B) 3

C) 4

D) 1

E) 2

Q
7

The number of x-intercepts of the graph of the function $y = -3\cot\left(2x + \frac{\pi}{2}\right)$ over the interval $[-\pi, \pi]$ is equal to

A) 3

B) 2

C) 4

D) 5

E) 1

Q
8

If $y = 2\cot 2x$, then the number of vertical asymptotes over the interval $\left(-\frac{\pi}{4}, \frac{3\pi}{4}\right)$ is equal to

A) 2

B) 1

C) 3

D) 4

E) 0

Q
9

The number of x-intercepts of the graph of $y = \frac{2}{3}\tan\left(\frac{3x}{4} - \pi\right)$ over the interval $\left(0, \frac{10\pi}{3}\right)$ is

A) 2

B) 1

C) 3

D) 4

E) 5

Q
10

The graph of $y = \cot\left(3x + \frac{\pi}{4}\right)$ over the interval $\left(-\frac{\pi}{12}, \frac{7\pi}{12}\right)$ intersects the x-axis at:

A) $\frac{\pi}{12}$ and $\frac{5\pi}{12}$

B) $-\frac{\pi}{8}$ and $\frac{\pi}{8}$

C) $-\frac{\pi}{8}$ and $\frac{5\pi}{12}$

D) $\frac{5\pi}{12}$ and $\frac{7\pi}{12}$

E) $\frac{\pi}{12}$ and $\frac{\pi}{8}$

Q
11

Over the interval $\left[-\frac{3\pi}{2},\ 3\pi\right]$, the graph of $y = \tan\frac{2x}{3}$ has

A) four vertical asymptotes

B) three vertical asymptotes

C) five x-intercepts

D) five vertical asymptotes

E) no y-intercept

Q
12

The number of vertical asymptotes of $y = 3 + 2\cot\frac{\pi\, x}{3}$ over the interval $[-4,\ 4]$ is

A) 6

B) 3

C) 2

D) 1

E) 4

Q
13

The number of vertical asymptotes of the graph of $y = -3\cot\left(\frac{2x}{3}\right)$ on the interval $\left[-\frac{3\pi}{4}, \frac{15\pi}{4}\right]$ is

A) 3

B) 2

C) 4

D) 5

E) 6

Q
14

The number of vertical asymptotes of $f(x) = 2\cot\left(\frac{3x}{2}\right)$ in the interval $\left(-\frac{\pi}{6}, 3\pi\right)$ is

A) 5

B) 9

C) 3

D) 2

E) 4

Topic 2 | Cosecant Graphs

Asymptotes, range, intercepts and monotonicity of cosecant graphs • Questions 15–26

Q **15**

For $-\frac{7\pi}{2} < x < 0$, the number of vertical asymptotes of the graph of $y = \csc\left(\frac{1}{2}x - \frac{\pi}{4}\right)$ is:

A) 1

B) 4

C) 2

D) 3

E) 6

Q **16**

Which one of the following statements is FALSE about the graph of $y = \frac{3}{2}\csc\left(x - \frac{\pi}{2}\right)$ in the interval $[-\pi, 2\pi]$?

A) the graph has four vertical asymptotes

B) the graph has a period of 2π

C) the graph has no x-intercept

D) the graph has one y-intercept

E) the range of the graph is $\left(-\infty, -\frac{3}{2}\right] \cup \left[\frac{3}{2}, \infty\right)$

Q
17

The graph of $y = -\csc(2x + \pi) + 2$, where $-\frac{3\pi}{4} \leq x \leq \frac{3\pi}{4}$, has

A) three x-intercepts.

B) four vertical asymptotes.

C) one y-intercept.

D) two vertical asymptotes.

E) four x-intercepts.

Q
18

The range of the graph of the function $f(x) = 1 - 2\csc x$ is

A) $(-\infty, -1] \cup [3, \infty)$

B) $(-\infty, -2] \cup [2, \infty)$

C) $(-\infty, -3] \cup [2, \infty)$

D) $[1, \infty)$

E) $[-1, 3]$

Q
19

The sum of all the vertical asymptotes of the graph of $y = -\csc\left(\frac{x}{3} - \frac{\pi}{6}\right)$ in the interval $[-4\pi, 2\pi]$ is

A) -2π

B) 4π

C) 2π

D) $-\pi$

E) π

Q
20

The number of vertical asymptotes of the graph of $y = \csc\left(\frac{1}{2}x - \frac{\pi}{4}\right)$ over the interval $\left(0, \frac{9\pi}{2}\right)$ is

A) 2

B) 1

C) 3

D) 4

E) 5

Q
21

The number of vertical asymptotes of the graph of the function $f(x) = 2 + 3\csc(2x - \pi)$ on the interval $[-\pi, \pi]$ is equal to

A) 5

B) 6

C) 4

D) 3

E) 2

Q
22

The number of vertical asymptotes of $y = \frac{3}{2}\csc\left(x - \frac{\pi}{2}\right)$ over the interval $\left(-\frac{3\pi}{2}, \frac{5\pi}{2}\right)$ is:

A) 3

B) 2

C) 4

D) 1

E) 5

Q
23

For $0 < x < \frac{2}{3}$, the graph of the function $y = 2\csc 3\pi x$ is decreasing on

A) $\left(0, \frac{1}{6}\right) \cup \left(\frac{1}{2}, \frac{2}{3}\right)$

B) $\left(\frac{1}{6}, \frac{1}{3}\right) \cup \left(\frac{1}{3}, \frac{1}{2}\right)$

C) $\left(0, \frac{1}{3}\right)$

D) $\left(\frac{1}{3}, \frac{2}{3}\right)$

E) $\left(\frac{1}{6}, \frac{1}{3}\right) \cup \left(\frac{1}{2}, \frac{2}{3}\right)$

Q
24

The graph of $y = -2\csc\frac{\pi x}{2}$, $-4 < x < 0$, is increasing on

A) $(-4, -3) \cup (-1, 0)$

B) $(-2, 0)$

C) $(-3, -1)$

D) $(-3, -2)$

E) $(-4, -2)$

Q
25

Let $f(x) = 1 + \csc\left(2x + \frac{\pi}{6}\right)$. Then which one of the following statements is TRUE?

A) the graph of f has infinitely many x-intercepts

B) the range of f is $(-\infty, -1] \cup [1, \infty)$

C) the period of f is 2π

D) the phase shift is $-\frac{\pi}{6}$

E) the graph of f has no y-intercept

Q
26

If $x = \frac{2\pi}{3}$ and $x = -\frac{4\pi}{3}$ are two consecutive vertical asymptotes for the graph of the function $f(x) = \csc\left(bx + \frac{2\pi}{3}\right) + 3$, $b > 0$, then $b =$

A) $\frac{1}{2}$

B) 1

C) 2

D) $\frac{1}{3}$

E) 3

Topic 3 | Secant Graphs

Range, asymptotes, sign and monotonicity of secant graphs • Questions 27–36

Q **27**

Which one of the following statements is TRUE?

A) The domain of $y = \sec(4x + \pi)$ is $x \neq \frac{n\pi}{4} - \frac{\pi}{8}$, where n is an integer.

B) $\sec x = -\sec(-x)$

C) The graph of $y = \sec x$ intersects the x-axis at $x = n\pi$, where n is an integer.

D) $\sec 30° = \frac{\sqrt{3}}{3}$

E) $\sec^2 x = 1 - \tan^2 x$

Q **28**

The graph of the function $f(x) = -2\sec\left(\pi x + \frac{\pi}{3}\right), -\frac{1}{3} < x < \frac{5}{3}$, is completely above the x-axis on the interval

A) $\left(\frac{1}{6}, \frac{7}{6}\right)$

B) $\left(-\frac{1}{3}, \frac{1}{6}\right)$

C) $\left(\frac{7}{6}, \frac{5}{3}\right)$

D) $\left(0, \frac{1}{6}\right)$

E) $\left(-\frac{1}{3}, \frac{1}{6}\right) \cup \left(\frac{7}{6}, \frac{5}{3}\right)$

Q
29

If $(a,\ b)$ is the minimum point on the graph of $f(x) = -2\sec\left(\frac{1}{2}\pi\ x + \pi\right)$, $3 < x < 5$, then $a + b =$

A) 6

B) 2

C) -2

D) π

E) 0

Q
30

If the range of the function $f(x) = -3\sec(2x + 1) + 2$ is $(-\infty,\ m] \cup [n,\ \infty)$, then $m + n =$

A) 4

B) 6

C) -4

D) 0

E) 3

Q
31

The graph of the function $f(x) = -\sec(\pi x)$, $-\frac{1}{2} < x < 1$, is increasing on the interval

A) $\left(0, \frac{1}{2}\right)$

B) $\left(-\frac{1}{2}, 0\right)$

C) $\left(-\frac{1}{2}, \frac{1}{2}\right)$

D) $(0, 1)$

E) $\left(\frac{1}{2}, 1\right)$

Q
32

On the interval $\left[-\frac{\pi}{4}, \frac{5\pi}{4}\right]$, if the graph of $y = 2 + 3\sec(2x - \pi)$ has four vertical asymptotes at $x = a_1, a_2, a_3$ and a_4, then $a_1 + a_2 + a_3 + a_4 =$

A) 2π

B) 3π

C) $\frac{5\pi}{2}$

D) $\frac{5\pi}{4}$

E) $\frac{9\pi}{4}$

Q
33

The range of the graph of $y = 2 - 2\sec(x + \pi)$ is

A) $(-\infty, 0] \cup [4, \infty)$

B) $(-\infty, -2] \cup [2, \infty)$

C) $(-\infty, 1] \cup [2, \infty)$

D) $[-2, 2]$

E) $[0, 4]$

Q
34

If P is the period of the graph of $f(x) = 5\sec\left(2\left(x - \frac{\pi}{4}\right)\right)$ and A is the amplitude of $y = -\pi\sin\left(\frac{x}{3}\right)$, then $A + P =$

A) 2π

B) 0

C) 4π

D) π

E) 3π

Q
35

In which one of the following intervals is the graph of $y = \sec\left(2x + \frac{\pi}{3}\right)$ below the x-axis?

A) $\left(\frac{\pi}{12}, \frac{7\pi}{12}\right)$

B) $\left(\frac{7\pi}{12}, \frac{13\pi}{12}\right)$

C) $\left(\frac{7\pi}{12}, \frac{5\pi}{6}\right)$

D) $\left(\frac{5\pi}{6}, \frac{13\pi}{12}\right)$

E) $\left(\frac{\pi}{3}, \frac{5\pi}{6}\right)$

Q
36

Which statement about the graph of the function $f(x) = -3\sec\left(\frac{\pi}{4}x\right)$ over the interval $\left[\frac{5}{2}, 5\right]$ is true?

A) There is a minimum but no maximum for the function.

B) The maximum value of the function is $f\left(\frac{5}{2}\right)$

C) The maximum value of the function is $f(5)$

D) The graph has neither a minimum nor a maximum for the function.

E) The maximum value of the function is $f(4)$

Topic 4 | Intersections with a Horizontal Line

Counting the points where a tangent, cosecant or secant graph meets a horizontal line • Questions 37–40

Q **37**

The graph of the function $f(x) = \csc\left(\frac{\pi x}{2}\right)$, $-2 < x < 2$, intersects the line $y = -2$ at

A) 2 points

B) 5 points

C) 1 point

D) 4 points

E) 3 points

Q **38**

For $0 < x < \frac{5\pi}{4}$, the line $y = -3$ intersects the graph of $y = \tan(2x - \pi)$ at:

A) 2 points.

B) 4 points.

C) 5 points.

D) one point.

E) 3 points.

Q
39

The graph of the function $f(x) = -\sec\left(\frac{\pi}{2}x\right)$, over the interval $[0, 4]$, intersects the line $y = 1$ at

A) 1 point

B) 2 points

C) 5 points

D) 3 points

E) 4 points

Q
40

The graph of the function $y = \left|3\sec\frac{2x}{3}\right|$ over the interval $\left[0, \frac{3\pi}{2}\right]$ intersects the line $y = 4$ at

A) 1 point

B) 2 points

C) 3 points

D) 4 points

E) no point

Topic 5 | Equations from Graphs

Choosing the equation of a tangent, cotangent, secant or cosecant graph from its picture • Questions 41–50

Q **41**

The graph in the adjacent figure represents part of the graph of

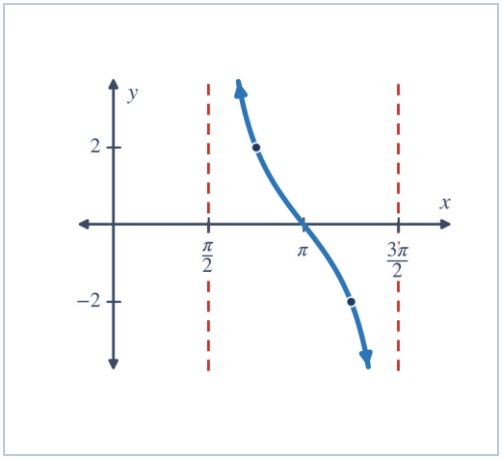
A) $y = -2\tan\left(x - \frac{3\pi}{2}\right)$

B) $y = 2\cot(x - \pi)$

C) $y = -2\tan\left(x - \frac{\pi}{2}\right)$

D) $y = 2\cot x$

E) $y = -2\tan x$



Q **42**

If $x = a$ and $x = b$ are two consecutive vertical asymptotes of the graph of $y = \frac{3}{2}\cot\left(3x + \frac{\pi}{4}\right)$ shown in the adjacent figure, then $a + b =$

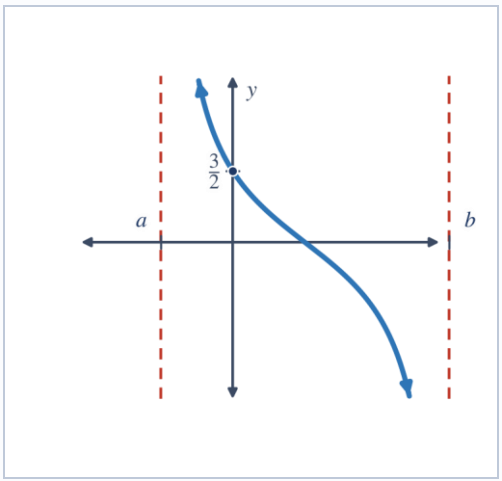
A) $\frac{\pi}{4}$

B) $\frac{\pi}{3}$

C) $\frac{\pi}{2}$

D) $-\frac{\pi}{12}$

E) $\frac{\pi}{6}$



Q
43

The equation of the adjacent graph is

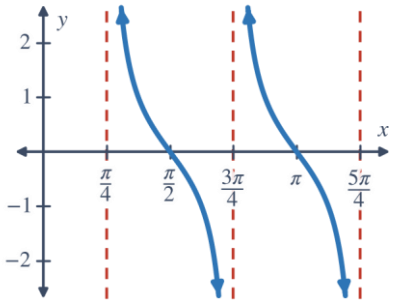
A) $y = \cot\left(2x - \frac{3\pi}{2}\right)$

B) $y = \tan\left(2x + \frac{3\pi}{2}\right)$

C) $y = \tan\left(2x + \frac{\pi}{2}\right)$

D) $y = \cot\left(x - \frac{\pi}{4}\right)$

E) $y = \cot\left(2x + \frac{\pi}{4}\right)$



Q
44

The adjacent figure represents part of the graph of

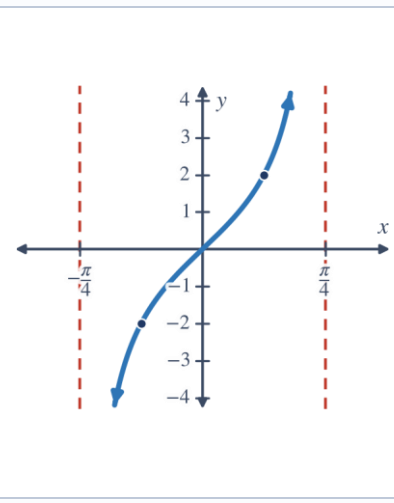
A) $y = -2\cot\left(2x + \frac{\pi}{2}\right)$

B) $y = -2\cot\left(x + \frac{\pi}{4}\right)$

C) $y = 2\tan\left(2x + \frac{\pi}{2}\right)$

D) $y = -2\tan(2x)$

E) $y = 2\tan\left(x + \frac{\pi}{4}\right)$



Q
45

The equation of the graph below is

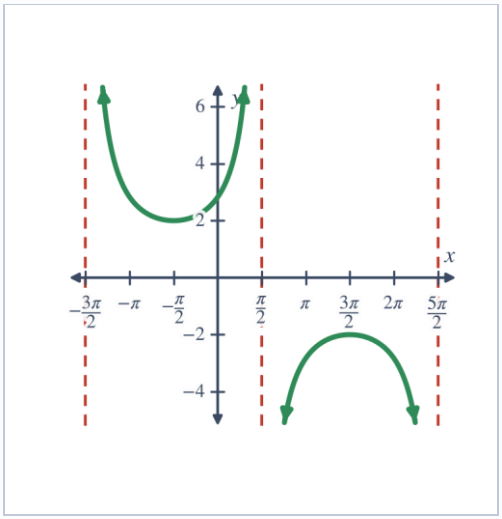
A) $y = 2\sec\left(2x - \frac{\pi}{4}\right)$

B) $y = -2\csc\left(2x - \frac{\pi}{4}\right)$

C) $y = 2\csc\left(\frac{1}{2}x - \frac{\pi}{4}\right)$

D) $y = -2\sec\left(\frac{1}{2}x - \frac{\pi}{4}\right)$

E) $y = -2\csc\left(\frac{1}{2}x - \frac{\pi}{4}\right)$



Q
46

The given graph in the adjacent figure represents part of the graph of the function

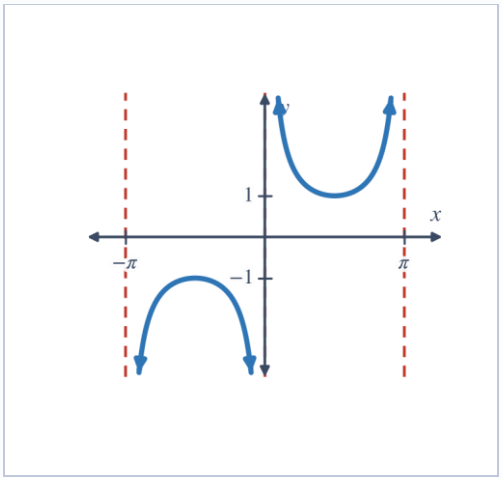
A) $y = \sec(x + \pi)$

B) $y = \csc\left(x + \frac{\pi}{2}\right)$

C) $y = \csc\left(x - \frac{\pi}{2}\right)$

D) $y = \sec(x - \pi)$

E) $y = \sec\left(x - \frac{\pi}{2}\right)$



Q
47

An equation for the given graph is

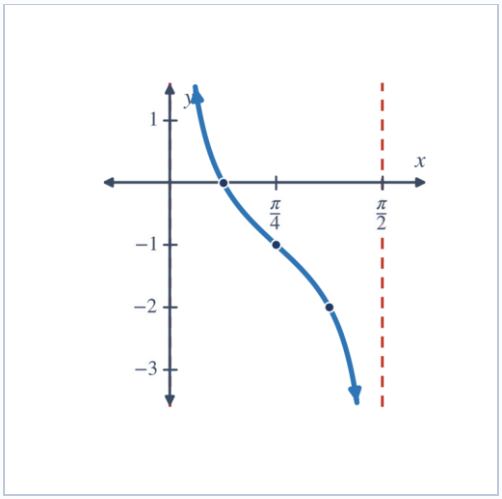
A) $y = -1 - \tan\left(2x - \frac{\pi}{2}\right)$

B) $y = -1 - \tan\left(x - \frac{\pi}{4}\right)$

C) $y = -1 + \cot\left(\frac{x}{2}\right)$

D) $y = 1 + \cot 2x$

E) $y = 1 - \cot\left(x - \frac{\pi}{4}\right)$



Q
48

If the adjacent figure represents the graph of $y = \cot\left(\frac{x}{2} - \frac{\pi}{4}\right)$, then $m =$

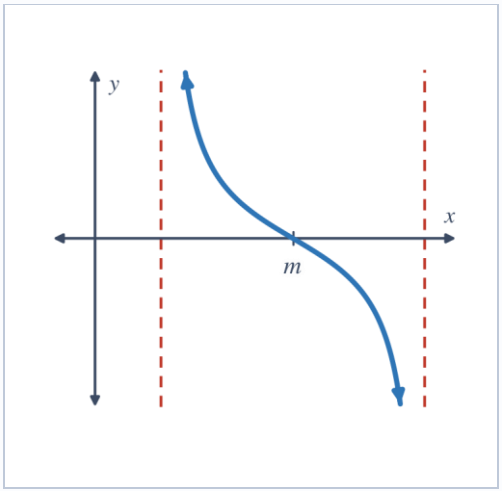
A) $\frac{3\pi}{2}$

B) $\frac{\pi}{2}$

C) $\frac{5\pi}{4}$

D) $\frac{3\pi}{4}$

E) $\frac{\pi}{4}$



Q
49

The figure below represents part of the graph of

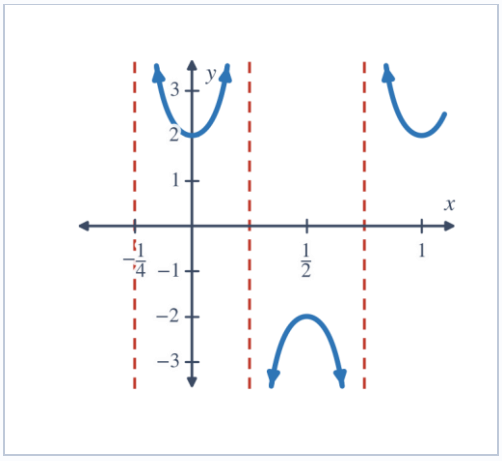
A) $y = 2\sec(2\pi\, x)$

B) $y = 2\csc(2\pi\, x)$

C) $y = 2\sec\left(\frac{\pi}{2}x\right)$

D) $y = -2\sec(2\pi\, x)$

E) $y = -2\csc\left(\frac{\pi}{2}x\right)$



Q
50

The adjacent graph represents part of the graph of

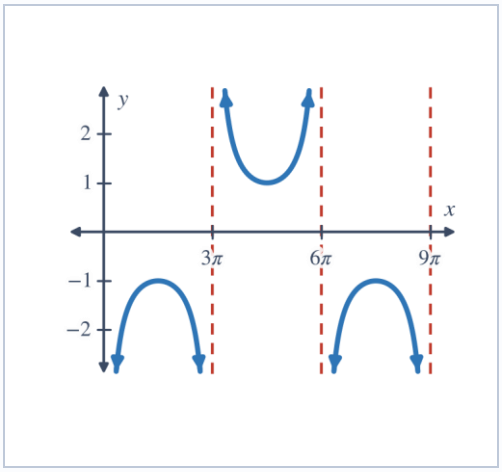
A) $y = \sec 9\, x$

B) $y = -\sec\frac{x}{3}$

C) $y = -\csc\frac{x}{3}$

D) $y = -\csc\frac{x}{9}$

E) $y = 2\csc\frac{x}{3}$



Topic 6 | Finding Constants, Periods and Phase Shifts

Finding the constants of a tangent or secant function and its period or phase shift • Questions 51–57

Q **51**

If the graph of the function $f(x) = a\tan(bx + c)$ has a period of $\frac{1}{4}$, a horizontal shift of $\frac{1}{2}$ to the right, and $f\left(\frac{7}{12}\right) = -\sqrt{3}$, then $\pi\ a + b =$

A) 3π

B) 5π

C) -2π

D) 7π

E) 2π

Q **52**

For the function $f(x) = m\sec(2x) + m^2$, where $m > 0$, if the range is $(-\infty, 2] \cup [6, \infty)$, then the value of m is equal to

A) 2

B) 1

C) 3

D) -3

E) -1

Q
53

If 2π is the period of the function $f(x) = a\tan(bx)$ and $f\left(\frac{\pi}{3}\right) = -\sqrt{3}$, then $a + b =$

A) $-\frac{5}{2}$

B) $\frac{5}{2}$

C) $-\frac{3}{2}$

D) $\frac{3}{2}$

E) $-\frac{7}{2}$

Q
54

If $f(x) = a\tan(bx)$, $b > 0$, is a tangent function with period 3 and $f(1) = 2\sqrt{3}$, then $f\left(\frac{3}{4}\right) =$

A) 2

B) $\sqrt{3}$

C) 1

D) $\frac{2\sqrt{3}}{3}$

E) $\frac{\sqrt{3}}{3}$

Q
55

The equation of a tangent function with period 2π and phase shift $\frac{\pi}{2}$ is

A) $y = \tan\left(\frac{x-\pi}{4}\right)$

B) $y = \tan\left(\frac{2x-\pi}{2}\right)$

C) $y = \tan\left(\frac{x-2\pi}{2}\right)$

D) $y = \tan\left(\frac{2x-\pi}{4}\right)$

E) $y = \tan\left(\frac{x-2\pi}{4}\right)$

Q
56

If $f(x) = -a \tan b\,x$, $a > 0$, $b > 0$, is a function of period 3, then $f\left(\frac{3}{4}\right)$ is

A) equal to $-\frac{a}{b}$

B) undefined

C) equal to $\frac{a}{b}$

D) equal to $-a$

E) equal to b

Q
57

The period P and the phase shift S of the function $f(x) = -2\sec\left(\frac{\pi}{3} - \frac{x}{4}\right) + 5$ are

A) $P = 2\pi,\ S = \frac{2\pi}{3}$

B) $P = 8\pi,\ S = \frac{4\pi}{3}$

C) $P = 4\pi,\ S = \frac{\pi}{3}$

D) $P = 8\pi,\ S = -\frac{4\pi}{3}$

E) $P = -8\pi,\ S = \frac{4\pi}{3}$

Answer Key

"Original" shows the question number in the source worksheet.

Q	Original	Ans.	Value
1	1	A	2
2	3	A	$\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right)$
3	4	A	$(0, 2) \cup (4, 6)$
4	5	A	6
5	10	A	4
6	13	E	2
7	15	D	5
8	16	A	2
9	37	A	2
10	42	A	$\frac{\pi}{12}$ and $\frac{5\pi}{12}$
11	48	B	three vertical asymptotes
12	52	B	3
13	57	A	3
14	59	A	5
15	17	A	1
16	18	A	the graph has four vertical asymptotes
17	20	A	three x-intercepts.
18	21	A	$(-\infty, -1] \cup [3, \infty)$
19	22	A	-2π
20	35	A	2
21	36	A	5
22	43	A	3
23	45	A	$\left(0, \frac{1}{6}\right) \cup \left(\frac{1}{2}, \frac{2}{3}\right)$
24	49	A	$(-4, -3) \cup (-1, 0)$
25	50	A	the graph of f has infinitely many x-intercepts
26	60	A	$\frac{1}{2}$
27	25	A	The domain of $y = \sec(4x + \pi)$ is $x \neq \frac{n\pi}{4} - \frac{\pi}{8}$, where n is an integer.
28	26	A	$\left(\frac{1}{6}, \frac{7}{6}\right)$
29	27	A	6
30	28	A	4
31	29	B	$\left(-\frac{1}{2}, 0\right)$
32	30	A	2π
33	31	A	$(-\infty, 0] \cup [4, \infty)$
34	34	A	2π
35	38	A	$\left(\frac{\pi}{12}, \frac{7\pi}{12}\right)$

Q	Original	Ans.	Value
36	58	**B**	The maximum value of the function is $f\left(\frac{5}{2}\right)$
37	23	**A**	2 points
38	24	**A**	2 points.
39	33	**A**	1 point
40	54	**B**	2 points
41	2	**E**	$y = -2\tan x$
42	11	**E**	$\frac{\pi}{6}$
43	12	**A**	$y = \cot\left(2x - \frac{3\pi}{2}\right)$
44	14	**A**	$y = -2\cot\left(2x + \frac{\pi}{2}\right)$
45	19	**E**	$y = -2\csc\left(\frac{1}{2}x - \frac{\pi}{4}\right)$
46	32	**E**	$y = \sec\left(x - \frac{\pi}{2}\right)$
47	39	**A**	$y = -1 - \tan\left(2x - \frac{\pi}{2}\right)$
48	41	**A**	$\frac{3\pi}{2}$
49	46	**A**	$y = 2\sec(2\pi\, x)$
50	53	**C**	$y = -\csc\frac{x}{3}$
51	6	**A**	3π
52	7	**A**	2
53	8	**A**	$-\frac{5}{2}$
54	9	**A**	2
55	47	**D**	$y = \tan\left(\frac{2x - \pi}{4}\right)$
56	51	**D**	equal to $-a$
57	55	**B**	$P = 8\pi,\ S = \frac{4\pi}{3}$