CHAPTER 6.5 & 5.4

Inverse Trigonometric Functions and Their Graphs

Multiple-Choice Old Exam Questions

37 questions • 4 topics • options A–E • two questions per page

Topic 1 | Exact Values of Inverse Trigonometric Expressions

Evaluating compositions and sums of inverse trigonometric values by using the principal-value ranges • Questions 1–14

Q **1**

$$\cos^{-1}\left(\cos\left(\frac{8\pi}{7}\right)\right) =$$

A) $\frac{4\pi}{7}$

B) $\frac{5\pi}{11}$

C) $\frac{8\pi}{7}$

D) $\frac{6\pi}{7}$

E) $\frac{3\pi}{7}$

Q **2**

$$\cos^{-1}\left(\sin\left(\frac{2\pi}{7}\right)\right) =$$

A) $\frac{3\pi}{14}$

B) $\frac{\pi}{14}$

C) $\frac{5\pi}{14}$

D) $\frac{2\pi}{7}$

E) $\frac{3\pi}{7}$

Q
3

The exact value of $\sin^{-1}\left(\sin\frac{7\pi}{6}\right)+\tan\left(\cos^{-1}\left(-\frac{1}{2}\right)\right)$ is

A) $-\frac{\pi}{6}-\sqrt{3}$

B) $\frac{7\pi}{6}+\frac{\sqrt{3}}{3}$

C) $\frac{\pi}{6}+\sqrt{3}$

D) $\frac{5\pi}{6}-1$

E) $-\frac{\pi}{6}+\sqrt{3}$

Q
4

$$\sin^{-1}\left(\cos\left(\frac{5\pi}{11}\right)\right)=$$

A) $\frac{\pi}{22}$

B) $\frac{5\pi}{11}$

C) $\frac{20\pi}{11}$

D) $\frac{\pi}{11}$

E) $\frac{19\pi}{22}$

Q
5

The exact value of $\cos^{-1}\left(\cos\frac{10\pi}{3}\right)$ is

A) $\frac{2\pi}{3}$

B) $\frac{\pi}{3}$

C) $\frac{4\pi}{3}$

D) $\frac{10\pi}{3}$

E) undefined

Q
6

The exact value of $\cos^{-1}\left(\cos\left(\frac{5\pi}{4}\right)\right)$ is

A) $\frac{3\pi}{4}$

B) $\frac{\pi}{4}$

C) $\frac{5\pi}{4}$

D) $-\frac{\pi}{4}$

E) undefined

Q
7

$$\cos^{-1}\left(-\frac{1}{2}\right) - \tan^{-1}\left(\frac{\sqrt{3}}{3}\right) =$$

A) $\frac{\pi}{2}$

B) $\frac{3\pi}{2}$

C) $\frac{\pi}{3}$

D) $\frac{5\pi}{6}$

E) $\frac{7\pi}{4}$

Q
8

$$\tan^{-1}\left(\tan\frac{3\pi}{4}\right) =$$

A) $-\frac{\pi}{4}$

B) $\frac{\pi}{4}$

C) $-\frac{3\pi}{4}$

D) $\frac{3\pi}{4}$

E) undefined

Q
9

$$\sin^{-1}\left(\sin\frac{3\pi}{5}\right)-\cos^{-1}\left(\cos\frac{3\pi}{5}\right)=$$

A) $\frac{6\pi}{5}$

B) $-\frac{6\pi}{5}$

C) $\frac{\pi}{5}$

D) π

E) $-\frac{\pi}{5}$

Q
10

$$\tan^{-1}\left(\tan\frac{3\pi}{4}\right)+\sin\left(\sin^{-1}\frac{\pi}{4}\right)=$$

A) 0

B) $-\frac{\pi}{2}$

C) $\frac{\pi}{2}$

D) $-\frac{\pi}{4}+\frac{\sqrt{2}}{2}$

E) π

Q
11

$$\sin^{-1}\left(\frac{3}{5}\right)+\cos^{-1}\left(-\frac{4}{5}\right)=$$

A) $\frac{7}{5}$

B) $\frac{\pi}{2}$

C) π

D) $-\frac{1}{5}$

E) $\frac{3\pi}{2}$

Q
12

$$\cos^{-1}\left(\sin\frac{\pi}{5}\right)=$$

A) $\frac{3\pi}{10}$

B) $\frac{3\pi}{20}$

C) $\frac{3\pi}{5}$

D) $\frac{\pi}{10}$

E) $\frac{2\pi}{5}$

Q
13

The value of $\cos^{-1}\frac{1}{2} + \sin^{-1}\left(\sin\frac{7\pi}{6}\right)$ is

A) $\frac{\pi}{6}$

B) $\frac{\pi}{2}$

C) $\frac{3\pi}{2}$

D) $\frac{4\pi}{3}$

E) π

Q
14

If $\arctan\frac{\sqrt{3}}{3} = x$ and $\arccos\left(-\frac{1}{2}\right) = y$, then $x + y =$

A) $\frac{5\pi}{6}$

B) π

C) $\frac{7\pi}{3}$

D) $\frac{4\pi}{3}$

E) $\frac{7\pi}{6}$

Topic 2 | Equations, Right Triangles and Identities

Solving equations with inverse trigonometric functions and evaluating expressions with right triangles • Questions 15–22

Q **15**

If $\sec^{-1} 2 + \cos^{-1} x = \frac{\pi}{2}$, then $x =$

A) $\frac{1}{2}$

B) $-\frac{1}{2}$

C) 1

D) $\frac{\sqrt{3}}{2}$

E) $-\frac{\sqrt{3}}{2}$

Q **16**

$$\csc\left(\frac{\pi}{2} + \sin^{-1}\left(-\frac{3}{5}\right)\right) =$$

A) $\frac{5}{4}$

B) $-\frac{3}{5}$

C) $\frac{3}{5}$

D) $-\frac{5}{4}$

E) $\frac{5}{3}$

Q
17

$$\tan\left(\sin^{-1}\left(-\frac{5}{13}\right)\right) =$$

A) $-\frac{5}{12}$

B) $-\frac{4}{5}$

C) $\frac{12}{5}$

D) $-\frac{12}{5}$

E) $\frac{5}{12}$

Q
18

The exact value of $\cos\left(\sin^{-1}\left(-\frac{3}{5}\right)\right)$ is equal to

A) $\frac{4}{5}$

B) $-\frac{4}{5}$

C) $\frac{3}{4}$

D) $-\frac{3}{4}$

E) $\frac{3}{5}$

Q
19

From the adjacent figure $\theta =$

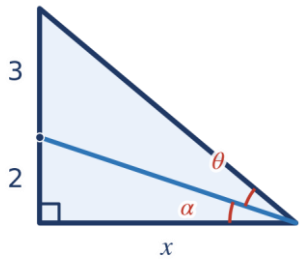
A) $\tan^{-1}\left(\frac{5}{x}\right) - \tan^{-1}\left(\frac{2}{x}\right)$

B) $\frac{1}{5}\tan^{-1}\left(\frac{5}{x}\right)$

C) $\tan^{-1}\left(\frac{x}{5}\right)$

D) $\tan^{-1}\left(\frac{2}{x}\right)$

E) $\tan^{-1}\left(\frac{3}{x}\right) - \tan^{-1}\left(\frac{5}{x}\right)$



Q
20

If $\arcsin\left(y - \frac{\pi}{3}\right) = \frac{\pi}{6}$, then y is equal to:

A) $\frac{3+2\pi}{6}$

B) $\frac{3-2\pi}{6}$

C) $\frac{5\pi}{6}$

D) $-\frac{3\sqrt{3}+2\pi}{6}$

E) $-\frac{3\sqrt{3}}{6}$

Q
21

The solution of the equation $\cos^{-1} x + \tan^{-1} \frac{5}{12} = \frac{\pi}{2}$ is

A) $\frac{5}{13}$

B) $-\frac{12}{13}$

C) $\frac{13}{12}$

D) $-\frac{13}{5}$

E) $\frac{7}{12}$

Q
22

The solution set of the equation $2\cos^{-1}\left(\frac{x-\pi}{3}\right) = 2\pi$ is

A) $\{\pi - 3\}$

B) $\{\pi + 2\}$

C) $\{4 - \pi\}$

D) $\{2\pi - 3\}$

E) $\{\pi\}$

Topic 3 | Statements and Angles from Ratios

True or false statements about inverse trigonometric functions, and finding an angle from two ratios • Questions 23–28

Q **23**

If $\sec\theta = -\frac{2\sqrt{3}}{3}$ and $\cot\theta = \sqrt{3}$, then a value of θ is

A) $210°$

B) $240°$

C) $225°$

D) $150°$

E) $330°$

Q **24**

If $\csc\theta = -\frac{2\sqrt{3}}{3}$ and $\cot\theta = \frac{\sqrt{3}}{3}$, then a value of θ is

A) $240°$

B) $210°$

C) $225°$

D) $330°$

E) $150°$

Q
25

Which one of the following statements is TRUE?

A) $\tan(\tan^{-1}(-11)) = -11$

B) $\tan^{-1}\left(\tan\frac{4\pi}{3}\right) = \frac{4\pi}{3}$

C) $\tan^{-1} x$ is an even function

D) $\tan^{-1} x = \frac{\sin^{-1} x}{\cos^{-1} x}$

E) $\tan^{-1}(-1) = \frac{\pi}{4}$

Q
26

Which one of the following statements is TRUE?

A) $\sin^{-1}\left(\frac{\pi}{5}\right)$ is defined

B) $\tan(\tan^{-1} 100)$ is undefined

C) $\cos\left(\cos^{-1}\left(-\frac{1}{2}\right)\right) = \frac{1}{2}$

D) $\tan^{-1} x = \frac{\sin^{-1} x}{\cos^{-1} x}$

E) The functions $y = \cos^{-1} x$ and $y = \sin^{-1} x$ have the same range

Q
27

Which one of the following statements is TRUE?

A) $\tan^{-1}\left(\tan\frac{4\pi}{3}\right) = \tan^{-1}\left(\tan\frac{\pi}{3}\right)$

B) $\sin^{-1}\left(\sin\frac{5\pi}{6}\right) = \frac{5\pi}{6}$

C) $\tan^{-1} x = \left(\frac{\sin x}{\cos x}\right)^{-1}$

D) $\cos^{-1}(\cos x) = x$ for $-1 \leq x \leq 1$

E) $y = \sin^{-1} x$ is an even function.

Q
28

Which of the following statements is FALSE?

A) the domain of $y = \cos^{-1}(x + 1)$ is $[0, 2]$

B) $\tan(\tan^{-1} x) = x$, for any real number x.

C) the range of $y = 2\sin^{-1} x$ is $[-\pi, \pi]$

D) $\sin^{-1} x + \sin^{-1}(-x) = 0, -1 \leq x \leq 1$

E) $\cos^{-1}\left(\cos\frac{x}{2}\right) = \frac{x}{2}$ if $0 \leq x \leq 2\pi$

Topic 4 | Domain and Range of Inverse Trigonometric Functions

Finding domains, ranges and unknown constants for transformed inverse trigonometric functions • Questions 29–37

Q
29

If $[-6,\ 2]$ is the domain of $f(x) = -3\sin^{-1}\left(\frac{x}{4} - k\right) - \frac{\pi}{2}$, then $4k =$

A) -2

B) -1

C) 4

D) -6

E) 2

Q
30

If the domain of $f(x) = \pi + 5\cos^{-1}\left(\frac{x}{2} + b\right)$ is $[3,\ 7]$, then $2b + 1 =$

A) -4

B) -5

C) 4

D) 5

E) 6

Q
31

The range of $y = -\cos^{-1}(1 - 3x) - \pi$ is

A) $[-2\pi, -\pi]$

B) $[\pi - 1, \pi + 1]$

C) $\left[-\frac{3\pi}{2}, -\frac{\pi}{2}\right]$

D) $\left[0, \frac{2}{3}\right]$

E) $\left[-\frac{3\pi}{2}, \frac{\pi}{2}\right]$

Q
32

The domain D and the range R of the function $f(x) = \pi + \cos^{-1}(x - 1)$ are

A) $D = [0, 2]$ and $R = [\pi, 2\pi]$

B) $D = [0, 2]$ and $R = [0, 2\pi]$

C) $D = [-1, 1]$ and $R = [\pi, 2\pi]$

D) $D = [-1, 1]$ and $R = [0, 2\pi]$

E) $D = [0, 2]$ and $R = [-\pi, \pi]$

Q
33

If the domain of the function $f(x) = \frac{\pi}{2} - 3\sin^{-1}(2x - 3)$ is $[m, n]$, then $m + n =$

A) 3

B) 4

C) 2

D) $-\pi$

E) π

Q
34

The range of the function $y = \frac{\pi}{2} + 2\cos^{-1}(x + 1)$ is:

A) $\left[\frac{\pi}{2}, \frac{5\pi}{2}\right]$

B) $[0, \pi]$

C) $[0, 2]$

D) $\left[\frac{\pi}{2} - 1, \frac{\pi}{2} + 1\right]$

E) $\left[-\frac{\pi}{2}, \frac{3\pi}{2}\right]$

Q
35

The graph of the function $y = \frac{1}{3}\tan^{-1} 2\,x - \frac{\pi}{3}$ has

A) range $= \left(-\frac{\pi}{2}, -\frac{\pi}{6}\right)$, domain $= (-\infty, \infty)$

B) range $= \left(\frac{\pi}{6}, \frac{\pi}{2}\right)$, domain $= (-\infty, \infty)$

C) range $= \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, domain $= (-\infty, \infty)$

D) range $= \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, domain $= [-1, 1]$

E) range $= (-\infty, \infty)$, domain $= \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Q
36

The domain of the function $y = \frac{\pi}{2} + 2\sin^{-1}\left(\frac{3}{2}x - \frac{5}{2}\right)$ is

A) $\left[1, \frac{7}{3}\right]$

B) $\left[\pi, \frac{7\pi}{3}\right]$

C) $\left(-\infty, \frac{-\pi+5}{3}\right] \cup \left[\frac{\pi+5}{3}, \infty\right)$

D) $\left[\frac{-\pi+5}{3}, \frac{\pi+5}{3}\right]$

E) $(-\infty, 1] \cup \left[\frac{7}{3}, \infty\right)$

Q
37

The range of the function $y = \frac{\pi}{3} + \frac{1}{2}\sin^{-1}\left(x - \frac{\pi}{3}\right)$ is:

A) $\left[\frac{\pi}{12}, \frac{7\pi}{12}\right]$

B) $\left[\frac{\pi}{3} - \frac{1}{2}, \frac{\pi}{3} + \frac{1}{2}\right]$

C) $\left(-\infty, \frac{\pi}{3} - \frac{1}{2}\right] \cup \left[\frac{\pi}{3} + \frac{1}{2}, \infty\right)$

D) $\left[-\frac{\pi}{3}, \frac{\pi}{3}\right]$

E) $\left(-\infty, \frac{\pi}{12}\right] \cup \left[\frac{7\pi}{12}, \infty\right)$

Answer Key

"Original" shows the question number in the source worksheet.

Q	Original	Ans.	Value
1	3	**D**	$\frac{6\pi}{7}$
2	4	**A**	$\frac{3\pi}{14}$
3	10	**A**	$-\frac{\pi}{6}-\sqrt{3}$
4	11	**A**	$\frac{\pi}{22}$
5	13	**A**	$\frac{2\pi}{3}$
6	15	**A**	$\frac{3\pi}{4}$
7	16	**A**	$\frac{\pi}{2}$
8	20	**A**	$-\frac{\pi}{4}$
9	23	**E**	$-\frac{\pi}{5}$
10	24	**A**	0
11	27	**C**	π
12	28	**A**	$\frac{3\pi}{10}$
13	29	**A**	$\frac{\pi}{6}$
14	31	**A**	$\frac{5\pi}{6}$
15	1	**D**	$\frac{\sqrt{3}}{2}$
16	8	**A**	$\frac{5}{4}$
17	17	**A**	$-\frac{5}{12}$
18	18	**A**	$\frac{4}{5}$
19	19	**A**	$\tan^{-1}\left(\frac{5}{x}\right)-\tan^{-1}\left(\frac{2}{x}\right)$
20	22	**A**	$\frac{3+2\pi}{6}$
21	32	**A**	$\frac{5}{13}$
22	34	**A**	$\{\pi-3\}$
23	5	**A**	$210°$
24	6	**A**	$240°$
25	7	**A**	$\tan(\tan^{-1}(-11))=-11$
26	9	**A**	$\sin^{-1}\left(\frac{\pi}{5}\right)$ is defined
27	21	**A**	$\tan^{-1}\left(\tan\frac{4\pi}{3}\right)=\tan^{-1}\left(\tan\frac{\pi}{3}\right)$
28	33	**A**	the domain of $y=\cos^{-1}(x+1)$ is $[0,\ 2]$

Q	Original	Ans.	Value
29	2	**A**	-2
30	12	**A**	-4
31	14	**A**	$[-2\pi, -\pi]$
32	25	**A**	$D = [0, 2]$ and $R = [\pi, 2\pi]$
33	26	**A**	3
34	30	**A**	$\left[\frac{\pi}{2}, \frac{5\pi}{2}\right]$
35	35	**A**	range $= \left(-\frac{\pi}{2}, -\frac{\pi}{6}\right)$, domain $= (-\infty, \infty)$
36	36	**A**	$\left[1, \frac{7}{3}\right]$
37	37	**A**	$\left[\frac{\pi}{12}, \frac{7\pi}{12}\right]$