4.4: (Laws of Logarithms)

Which one of the following statements is TRUE?

A) $\frac{\log_{2a} 16}{\log_{2a} 4} = 2$ for any real number $a > 0$, and $a \neq \frac{1}{2}$.

B) $\log(x + y) = \log x + \log y$ for all positive real numbers x and y.

C) $\log(5x) - \log(2x) = \log(3x)$ for all real numbers $x > 0$.

D) $\log(2x) = 2\log x$ for all real numbers $x > 0$.

E) $\log x^2 = 2\log x$ for all real numbers x.

Which one of the following statements is TRUE?

A) $\ln x > 0$, if $x > 1$

B) $\frac{\ln x}{\ln y} = \ln \frac{x}{y}, x > 0, y > 0$

C) $\ln x^2 = 2\ln x$, for any real number x

D) $e^{\ln x} = e^x$

E) $\ln(x + y) = \ln x + \ln y, x > 0, y > 0$

Let $a > 1$ and $y > 0$. If $\log_8 a = x + 1$ and $\log_a y = \frac{1}{3}$, then $2^x =$

A) y

B) $y - 2$

C) $\frac{y}{2}$

D) $2y$

E) y^2

If $8 \log_{25} \sqrt[4]{125} + \frac{\ln 2}{\ln 5} - 5^{\log_{25} 9} = \log_5 A$, then $A =$

A) 5

B) 2

C) 1

D) 8

E) 4

$(\sqrt{10})^{2\log 2} + \log_{\frac{2}{5}}\left(\frac{4}{25}\right) =$

A) 4

B) $\frac{1}{4}$

C) 6

D) 8

E) 2

The x-intercept of the graph of the function $f(x) = e^{2\ln 3} + \ln e^{(\ln x - \ln 4)}$ is

A) $4e^{-9}$

B) $9e^{-4}$

C) $-4e^{-9}$

D) $4e^{9}$

E) $-8\ln 3$

If $\log_3(x+1) = \frac{1}{2}$, then $\log_3(3x^2+6x+3) =$

A) 2

B) $\frac{3}{2}$

C) 4

D) $\frac{7}{2}$

E) $\frac{5}{2}$

If $\log_3(5) = y$, then $\log_5(9) =$

A) $\frac{2}{y}$

B) $\frac{3}{y}$

C) $3y$

D) $2y$

E) y^2

If $\frac{1}{x}\log_2(e) = y$, then $\ln 8 =$

A) $\frac{3}{xy}$

B) $\frac{3y}{x}$

C) $\frac{3x}{y}$

D) $\frac{x}{3y}$

E) $3xy$

Let $x > 1$. If $\log_{0.5}(x-1) = a$, then $\log_8(2x^2 - 4x + 2)$

A) $\frac{1+2a}{3}$

B) $\frac{2a-1}{3}$

C) $1 - 2a$

D) $\frac{1-2a}{3}$

E) $\frac{1-2a}{6}$

If x, y and z are positive real numbers, then, $\log_5 x + 4\log_{25} y - 3\log_5 z =$

A) $\log_5 \frac{x^2y^4}{3z}$

B) $\log_5 \frac{xy^8}{z^6}$

C) $\log_5 \frac{x^2y^4}{z^3}$

D) $\log_5 \frac{xy^4}{z^6}$

E) $\log_5 \frac{xy^2}{z^3}$

$\left(\log_{\sqrt{2}} 8\right)\left(\log_{32} \sqrt[3]{25}\right)\left(\log_{5^{-2}} 4\right) =$

A) $-\frac{4}{5}$

B) $\frac{4}{5}$

C) $-\frac{1}{3}$

D) $\frac{1}{3}$

E) -3

$\left(\frac{1}{10}\right)^{\log 3} + \log_{\frac{3}{2}}\left(\frac{8}{27}\right) =$

A) $-\frac{8}{3}$

B) 0

C) $\frac{10}{3}$

D) 6

E) $-\frac{2}{3}$

If $A = 2^{\log_8 125}$ and $B = \left(\log_{\sqrt{2}} 9\right)\left(\log_3 \sqrt{8}\right)$, then $B + A =$

A) 11

B) 1

C) 10

D) 0

E) 12

Which one of the following statements is always TRUE for the real numbers $x > 0, y > 0, x \neq 1$ and $y \neq 1$?

A) $\frac{\ln x}{\ln y} = -\frac{\log_x x}{\log_x \frac{1}{y}}$

B) $(\log_y x)(\log_x y) = -1$

C) $(\log_y x)^n = n\log_y x$

D) $\log_x \frac{1}{y} = \log_y x$

E) $\log_x(x + y^2) = 1 + 2\log_x y$

Which one of the following statements is TRUE for all $x > 0, y > 0, b > 0$ and $b \neq 1$?

A) $\log_b \sqrt{x} = \frac{\ln x}{2\ln b}$

B) $\log_b(x + y) = \log_b x + \log_b y$

C) $(\log_b x)(\log_b y) = \log_b(xy)$

D) $\log_b \left(\frac{x}{y}\right) = \frac{\log_b x}{\log_b y}, y \neq 1$

E) $\frac{\log_b x}{\log_b y} = \log_b x - \log_b y, y \neq 1$

If $\log_3(5) = a$ and $\log_3(2) = b$, then $\log_{\sqrt{2}}(30) =$

A) $\frac{2+2a+2b}{b}$

B) $\frac{2a+2b+2b^2}{b}$

C) $\frac{\sqrt{1+a+b}}{b}$

D) $\frac{2a}{b^2}$

E) $2 + 2a + 2b$

If $\log 5 = a,\ \log 3 = b$, then $\log_3(45) =$

A) $\frac{a+2b}{a}$

B) $\frac{2a+2b}{b}$

C) $\frac{a+2b}{b}$

D) $\frac{2a+b}{b}$

E) $\frac{a+b}{2b}$

If $\log_6 3 = a$, then $\log_2 108 =$

A) $\frac{a+2}{1-a}$

B) $\frac{a-2}{1-a}$

C) $\frac{a+3}{1-a}$

D) $\frac{a-3}{1-a}$

E) $\frac{a-2}{1+a}$

If $\log_6 2 = x$, then $\frac{1}{2}\log_2 144 =$

A) $\frac{x+1}{x}$

B) $x + 1$

C) $\frac{x}{x-1}$

D) $\frac{1}{x}$

E) $\sqrt{x}$

If $(a, 0)$ is the x-intercept and $(0, b)$ is the y - intercept of the function $f(x) = \log\left(\frac{1}{2}\right) + \log(20 - 2x)$, then $a + b =$

A) 10

B) 8

C) -8

D) -10

E) 12

If $x > 0,\ x \neq 1,\ y > 0, \ln x = u$ and $\ln y = v$, then the expression $\log_x\left(\sqrt[3]{x}y^4\right)$ simplifies to

A) $\frac{1}{3} + 4\frac{v}{u}$

B) $3 + 4\frac{u}{v}$

C) $\frac{1}{3}u + 4v$

D) $3u + 4v$

E) $\frac{1}{3}u + \frac{4}{3}v$

If $\log 2 = t$, then $\log 800 - \log\left(\frac{1}{25}\right) =$

A) $t + 4$

B) $t + 2$

C) $5t + 4$

D) $5t + 2$

E) $2t + 3$

The exact value of the expression $\left(\log_{49} \sqrt[3]{7} + \sqrt{\log_{0.5} \frac{1}{16}}\right)$ is equal to

A) $\frac{13}{6}$

B) $\frac{25}{6}$

C) $\frac{7}{3}$

D) $\frac{7}{2}$

E) $\frac{2}{3}$

If the expression $-1 + \log_{16} x^3y^4 + \log_{\frac{1}{8}} x^4y^3$ where $x > 0,\ y > 0$, is written as a single logarithm with base 2, then it is equal to

A) $\log_2\left(\frac{1}{2x^{7/12}}\right)$

B) $\log_2\left(\frac{y}{2x^{5/12}}\right)$

C) $\log_2\left(\frac{x^{7/12}}{2y}\right)$

D) $\log_2\left(-x^{5/12}\right)$

E) $\log_2\left(-x^{5/12}y\right)$

The exact value of the expression $\left(\log_5 \sqrt[4]{25} + \log_{0.01} 1000\right)$ is equal to

A) -1

B) $\frac{13}{2}$

C) $-\frac{11}{2}$

D) $-\frac{5}{2}$

E) -2

If $\log 2 = x$ then $\log 1600 + \log \frac{1}{5}$ is equal to

A) $5x - 1$

B) $3x + 3$

C) $3x$

D) $5x + 1$

E) $5x - 2$

The expression $1 + 2\ln x - \frac{\ln(x+1)}{2} + \log_{\sqrt{e}} \sqrt{5}$ can be written as

A) $\ln\left(\frac{5ex^2}{\sqrt{x+1}}\right)$

B) $\ln\left(\frac{\sqrt{5e}x^2}{\sqrt{x+1}}\right)$

C) $\ln\left(25x^2\sqrt{x+1}\right)$

D) $\ln\left(\frac{5x}{\sqrt{x+1}}\right)$

E) $\ln\left(\frac{5ex^2}{x+1}\right)$

If the expression $-1 + 6\log_{\frac{1}{8}}(wx) - 4\log_{\sqrt{2}}\left(\frac{1}{y}\right)$ is written as a single logarithm with base 2, then it is equal to

A) $\log_2 \frac{3(wx)^2}{2y^4}$

B) $\log_2 \frac{y^8}{2(wx)^2}$

C) $\log_2 \frac{2y^4}{(wx)^2}$

D) $\log_2 \frac{y^2}{2(wx)^2}$

E) $\log_1 \frac{\mathbf{2y^2}}{\mathbf{(wx)^2}}$

If $\log 2 = a$ and $\log 3 = b$, then $\frac{\log_2 \frac{9}{2}}{\log_2 10}$ in terms of a and b is equal to

A) $2b - a$

B) $\frac{a+b}{2}$

C) $\frac{2}{ab}$

D) $\frac{ab}{2}$

E) $a - 2b$

$\log_{\frac{3}{2}}\left(\frac{8}{27}\right) + \sqrt{3}^{\left(\frac{\log 4}{\log 3}\right)} =$

A) 0

B) -1

C) 1

D) 2

E) -2

If $\log_3 2 = a$ and $\log_3 5 = b$, then $\log 9$ in terms of a and b is equal to

A) $\frac{2}{a+b}$

B) $\frac{1}{a} + \frac{2}{b}$

C) $\frac{ab}{2}$

D) $\frac{a+b}{2}$

E) $\frac{2}{ab}$

If $\ln 2 = x$ and $\ln 6 = y$, then $\log_9(12) =$

A) $\frac{x+y}{2y-2x}$

B) $\frac{x+y}{y+2x}$

C) $\frac{x-y}{2y+2x}$

D) $\frac{x+y}{x-y}$

E) $\frac{x-y}{x+y}$

If $x > 0, y > 0$ and $w > 0$, then $\log(y^3w^2) - 3\log(x\sqrt{y}) + 2\log\frac{x}{w} =$

A) $\log\frac{y\sqrt{y}}{x}$

B) $\log\frac{y\sqrt{y}}{w}$

C) $\log\frac{y}{x}$

D) $\log\frac{\sqrt{y}}{x}$

E) $\log\frac{y}{w}$

If $m > 0$, the expression $-\frac{2}{3}\log_5(5m^2) + \frac{1}{2}\log_5(25m^2) + \log_5 \sqrt[3]{\frac{m}{5}}$

A) 0

B) 1

C) 5

D) $\frac{5}{m}$

E) $\frac{m}{5}$

If $\log 2 = x$ and $\log 6 = y$, then $\log 15 =$

(A) $y - 2x + 1$

B) $2y - x + 1$

C) $2y - x + 2$

D) $\frac{y-x}{x}$

E) $\frac{y+x}{x}$

If $\ln(4) = x$ and $\ln(5) = y$, then $\log_{\frac{4}{5}}\left(\frac{16e^2}{125}\right)$ can be written in terms of x and y as:

A) $\frac{2x+2-3y}{x-y}$

B) $\frac{2x-2-3y}{x-y}$

C) $\frac{2x-2+3y}{x-y}$

D) $\frac{2x+2-3y}{x+y}$

E) $\frac{2x-2-3y}{x+y}$

The expression $-\frac{2}{3}\log_7(5m^2) + \frac{1}{2}\log_7(25m^2) + \log_7 \sqrt[4]{25}$ where $m > 0$ is equal to

(A) $\log_7 \frac{5^{5/6}}{m^{1/3}}$

B) $-\log_7 m^{1/3}$

C) $-\log_7 m^3$

D) $-\log_7 \frac{5^{1/3}}{m^{56}}$

E) $\log_7 \frac{m^{1/3}}{5^{5/6}}$

If $A = (\sqrt[3]{2})^{\log_2 27}$ and $B = (\log_2 81) \cdot \left(\log_{\sqrt{3}} 16\right)$, then $B - A =$

A) 29

B) -5

C) -23

D) -29

E) 5

If $\log 3 = t$ and $\log 2 = x$, then $2 + \log(0.09) + \log 5 =$

A) $2t - x + 1$

B) $3t + x$

C) $3t - x$

D) $2t + 1$

E) $2t + x + 1$

Which one of the following statements is TRUE?

A) $y = \log_{(a-1)} x$ is defined if $a > 1, a \neq 2$, and $x > 0$
B) $\log(abc) = (\log a)(\log b)(\log c)$
C) $\log(a + b + c) = \log(a) + \log(b) + \log(c)$
D) $\ln 7 - \ln 2 = \frac{\ln 7}{\ln 2}$
E) $\ln x^2 = 2\ln x$, for any real number x

If $A = (\log_5 81)(\log_3 5)\left(\log_2 \sqrt{a}\right)$ and $B = e^{-\ln(1/4)}$, then $B^A =$

A) a^4
B) a^8
C) a
D) $-a$
E) a^{-1}

If $\log 0.4 = x$, then $\log_2 20 =$

A) $\frac{x+3}{x+1}$

B) $x + 1$

C) $\frac{x+1}{2}$

D) $x - 1$

E) $\frac{x+1}{x-1}$

If $x > 0$ and $\frac{\log_{\sqrt{5}}(4)}{\log_{25}(x^2)} = 2\log 10$, then $x =$

A) 4

B) 2

C) 1

D) $2\ln 2$

E) $\frac{1}{2}$

If $A = \frac{\log_{\sqrt{5}} 8}{\log_{25} 100}$, then the value of 10^A is equal to

(a) 64

(b) -4

(c) 3

(d) 32

(e) 6

Which one of the following statements is TRUE?

(a) $\log\left(x + \sqrt{x^2 - 1}\right) = -\log\left(x - \sqrt{x^2 - 1}\right)$ for any $x \geq 1$ (correct)

(b) $(\log x)^3 = 3\log x$ for any $x > 1$

(c) The equation $e^x = -1$ has a real solution

(d) $f(x) = \ln x - \ln(x - 2)$ is defined on $(-\infty, 0) \cup (2, \infty)$

(e) $\ln(x - 1) < 0$ when $x < 1$