CHAPTER 4.4

Laws of Logarithms

Multiple-Choice Old Exam Questions

49 questions • 5 topics • options A–E • two questions per page

Topic 1 | Statements about Logarithms

Deciding which statement about logarithms and the laws of logarithms is true • Questions 1–6

Q 1

Which one of the following statements is TRUE?

A) $\frac{\log_{2a} 16}{\log_{2a} 4} = 2$ for any real number $a > 0$ with $a \neq \frac{1}{2}$.

B) $\log(x + y) = \log\ x + \log\ y$ for all positive real numbers x and y.

C) $\log(5x) - \log(2x) = \log(3x)$ for all real numbers $x > 0$.

D) $\log(2x) = 2\log\ x$ for all real numbers $x > 0$.

E) $\log x^2 = 2\log\ x$ for all real numbers x.

Q 2

Which one of the following statements is TRUE?

A) $\ln\ x > 0$ if $x > 1$.

B) $\frac{\ln\ x}{\ln\ y} = \ln\frac{x}{y}, x > 0, y > 0$.

C) $\ln x^2 = 2\ln\ x$ for any real number x.

D) $e^{\ln\ x} = e^x$.

E) $\ln(x + y) = \ln\ x + \ln\ y, x > 0, y > 0$.

Q
3

Which one of the following statements is always TRUE for real numbers $x > 0, y > 0, x \neq 1$ and $y \neq 1$?

A) $\frac{\ln x}{\ln y} = -\frac{\log_x x}{\log_x \frac{1}{y}}$

B) $(\log_y x)(\log_x y) = -1$

C) $(\log_y x)^n = n\log_y x$

D) $\log_x \frac{1}{y} = \log_y x$

E) $\log_x(x + y^2) = 1 + 2\log_x y$

Q
4

Which one of the following statements is TRUE for all $x > 0, y > 0, b > 0$ and $b \neq 1$?

A) $\log_b \sqrt{x} = \frac{\ln x}{2\ln b}$

B) $\log_b(x + y) = \log_b x + \log_b y$

C) $(\log_b x)(\log_b y) = \log_b(xy)$

D) $\log_b\left(\frac{x}{y}\right) = \frac{\log_b x}{\log_b y}, y \neq 1$

E) $\frac{\log_b x}{\log_b y} = \log_b x - \log_b y, y \neq 1$

Q 5

Which one of the following statements is TRUE?

A) $y = \log_{(a-1)} x$ is defined if $a > 1$, $a \neq 2$ and $x > 0$.

B) $\log(abc) = (\log\ a)(\log\ b)(\log\ c)$.

C) $\log(a + b + c) = \log\ a + \log\ b + \log\ c$.

D) $\ln 7 - \ln 2 = \frac{\ln 7}{\ln 2}$.

E) $\ln x^2 = 2\ln\ x$ for any real number x.

Q 6

Which one of the following statements is TRUE?

A) $\log\left(x + \sqrt{x^2 - 1}\right) = -\log\left(x - \sqrt{x^2 - 1}\right)$ for any $x \geq 1$.

B) $(\log\ x)^3 = 3\log\ x$ for any $x > 1$.

C) The equation $e^x = -1$ has a real solution.

D) $f(x) = \ln\ x - \ln(x - 2)$ is defined on $(-\infty, 0) \cup (2, \infty)$.

E) $\ln(x - 1) < 0$ when $x < 1$.

Topic 2 | Exact Values of Logarithmic Expressions

Evaluating logarithmic and exponential expressions exactly • Questions 7–18

Q 7

If $8\log_{25}\sqrt[4]{125}+\frac{\ln 2}{\ln 5}-5^{\log_{25}9}=\log_5 A$, then $A=$

A) 5

B) 2

C) 1

D) 8

E) 4

Q 8

$$\left(\sqrt{10}\right)^{2\log 2}+\log_{2/5}\left(\frac{4}{25}\right)=$$

A) 4

B) $\frac{1}{4}$

C) 6

D) 8

E) 2

Q
9

$$\left(\log_{\sqrt{2}} 8\right)\left(\log_{32} \sqrt[3]{25}\right)\left(\log_{5^{-2}} 4\right) =$$

A) $-\frac{4}{5}$

B) $\frac{4}{5}$

C) $-\frac{1}{3}$

D) $\frac{1}{3}$

E) -3

Q
10

$$\left(\frac{1}{10}\right)^{\log 3} + \log_{3/2}\left(\frac{8}{27}\right) =$$

A) $-\frac{8}{3}$

B) 0

C) $\frac{10}{3}$

D) 6

E) $-\frac{2}{3}$

Q
11

If $A = 2^{\log_8 125}$ and $B = \left(\log_{\sqrt{2}} 9\right)\left(\log_3 \sqrt{8}\right)$, then $B + A =$

A) 11

B) 1

C) 10

D) 0

E) 12

Q
12

The exact value of the expression $\log_{49} \sqrt[3]{7} + \sqrt{\log_{0.5} \frac{1}{16}}$ is equal to

A) $\frac{13}{6}$

B) $\frac{25}{6}$

C) $\frac{7}{3}$

D) $\frac{7}{2}$

E) $\frac{2}{3}$

Q
13

The exact value of the expression $\log_5 \sqrt[4]{25} + \log_{0.01} 1000$ is equal to

A) -1

B) $\frac{13}{2}$

C) $-\frac{11}{2}$

D) $-\frac{5}{2}$

E) -2

Q
14

$$\log_{3/2}\left(\frac{8}{27}\right) + (\sqrt{3})^{\left(\frac{\log 4}{\log 3}\right)} =$$

A) 0

B) -1

C) 1

D) 2

E) -2

Q
15

If $m > 0$, the expression $-\frac{2}{3}\log_5(5m^2) + \frac{1}{2}\log_5(25m^2) + \log_5 \sqrt[3]{\frac{m}{5}}$ is equal to

A) 0

B) 1

C) 5

D) $\frac{5}{m}$

E) $\frac{m}{5}$

Q
16

If $A = \left(\sqrt[3]{2}\right)^{\log_2 27}$ and $B = (\log_2 81) \cdot \left(\log_{\sqrt{3}} 16\right)$, then $B - A =$

A) 29

B) -5

C) -23

D) -29

E) 5

Q
17

If $A = \dfrac{\log_{\sqrt{5}} 8}{\log_{25} 100}$, then the value of 10^A is equal to

A) 64

B) -4

C) 3

D) 32

E) 6

Q
18

If $\left(\frac{1}{9}\right)^{\log_3 2} + \log_{1/4}\left(\frac{9}{16}\right) = \frac{Q}{R} - \log_R(Q)$, then $R - Q =$

A) -5

B) -3

C) 3

D) 8

E) 1

Topic 3 | Combining and Simplifying Logarithmic Expressions

Writing a logarithmic expression as a single logarithm or simplifying it • Questions 19–26

Q **19**

If x, y and z are positive real numbers, then $\log_5 x + 4\log_{25} y - 3\log_5 z =$

A) $\log_5 \frac{x^2y^4}{3z}$

B) $\log_5 \frac{xy^8}{z^6}$

C) $\log_5 \frac{x^2y^4}{z^3}$

D) $\log_5 \frac{xy^4}{z^6}$

E) $\log_5 \frac{xy^2}{z^3}$

Q **20**

If $x > 0, x \neq 1, y > 0, \ln\ x = u$ and $\ln\ y = v$, then the expression $\log_x\left(\sqrt[3]{x}\, y^4\right)$ simplifies to

A) $\frac{1}{3} + 4\frac{v}{u}$

B) $3 + 4\frac{u}{v}$

C) $\frac{1}{3}u + 4v$

D) $3u + 4v$

E) $\frac{1}{3}u + \frac{4}{3}v$

Q
21

If the expression $-1+\log_{16}(x^3y^4)+\log_{1/8}(x^4y^3)$, where $x>0$ and $y>0$, is written as a single logarithm with base 2, then it is equal to

A) $\log_2\left(\frac{1}{2x^{7/12}}\right)$

B) $\log_2\left(\frac{y}{2x^{5/12}}\right)$

C) $\log_2\left(\frac{x^{7/12}}{2y}\right)$

D) $\log_2(-x^{5/12})$

E) $\log_2(-x^{5/12}y)$

Q
22

The expression $1+2\ln\ x-\frac{\ln(x+1)}{2}+\log_{\sqrt{e}}\sqrt{5}$ can be written as

A) $\ln\left(\frac{5ex^2}{\sqrt{x+1}}\right)$

B) $\ln\left(\frac{\sqrt{5e}\,x^2}{\sqrt{x+1}}\right)$

C) $\ln\left(25x^2\sqrt{x+1}\right)$

D) $\ln\left(\frac{5x}{\sqrt{x+1}}\right)$

E) $\ln\left(\frac{5ex^2}{x+1}\right)$

Q
23

If the expression $-1+6\log_{1/8}(wx)-4\log_{\sqrt{2}}\left(\frac{1}{y}\right)$ is written as a single logarithm with base 2, then it is equal to

A) $\log_2 \frac{3(wx)^2}{2y^4}$

B) $\log_2 \frac{y^8}{2(wx)^2}$

C) $\log_2 \frac{2y^4}{(wx)^2}$

D) $\log_2 \frac{y^2}{2(wx)^2}$

E) $\log_2 \frac{2y^2}{(wx)^2}$

Q
24

If $x>0, y>0$ and $w>0$, then $\log(y^3w^2)-3\log\left(x\sqrt{y}\right)+2\log\frac{x}{w}=$

A) $\log\frac{y\sqrt{y}}{x}$

B) $\log\frac{y\sqrt{y}}{w}$

C) $\log\frac{y}{x}$

D) $\log\frac{\sqrt{y}}{x}$

E) $\log\frac{y}{w}$

Q
25

If $m > 0$, the expression $-\frac{2}{3}\log_7(5m^2) + \frac{1}{2}\log_7(25m^2) + \log_7 \sqrt[4]{25}$ is equal to

A) $\log_7 \frac{5^{5/6}}{m^{1/3}}$

B) $-\log_7 m^{1/3}$

C) $-\log_7 m^3$

D) $-\log_7 \frac{5^{1/3}}{m^{56}}$

E) $\log_7 \frac{m^{1/3}}{5^{5/6}}$

Q
26

If $x > 0$ and $x \neq 1$, then $\log_x 4 - \log_{1/x} \sqrt{x^3} - \log_x \left(\frac{1}{x}\right) + \log_{\sqrt{x}}(2) =$

A) $\frac{5}{2} + 4\log_x 2$

B) $\frac{5}{2} + 2\log_x 2$

C) $\frac{7}{2} + 4\log_x 2$

D) $\frac{7}{2} + 2\log_x 2$

E) $\frac{3}{2} + 2\log_x 2$

Topic 4 | Solving Equations and Finding Intercepts

Using the laws of logarithms to solve an equation, find an intercept, or evaluate an expression • Questions 27–32

Q **27**

The x-intercept of the graph of the function $f(x) = e^{2\ln 3} + \ln e^{\ln x - \ln 4}$ is

A) $4e^{-9}$

B) $9e^{-4}$

C) $-4e^{-9}$

D) $4e^{9}$

E) $-8\ln 3$

Q **28**

If $\log_3(x+1) = \frac{1}{2}$, then $\log_3(3x^2 + 6x + 3) =$

A) 2

B) $\frac{3}{2}$

C) 4

D) $\frac{7}{2}$

E) $\frac{5}{2}$

Q
29

If $(a,\ 0)$ is the x-intercept and $(0,\ b)$ is the y-intercept of the function $f(x) = \log\left(\frac{1}{2}\right) + \log(20 - 2x)$, then $a + b =$

A) 10

B) 8

C) -8

D) -10

E) 12

Q
30

If $A = (\log_5 81)(\log_3 5)\left(\log_2 \sqrt{a}\right)$ and $B = e^{-\ln(1/4)}$, then $B^A =$

A) a^4

B) a^8

C) a

D) $-a$

E) a^{-1}

Q
31

If $x > 0$ and $\frac{\log_{\sqrt{5}}(4)}{\log_{25}(x^2)} = 2\log 10$, then $x =$

A) 4

B) 2

C) 1

D) $2\ln 2$

E) $\frac{1}{2}$

Q
32

If $\log_b 2 = -\log_b a$ and $\log_4 y = \log_a 2$, then $y =$

A) $\frac{1}{4}$

B) 4

C) $\frac{1}{8}$

D) $\frac{\sqrt{2}}{2}$

E) 8

Topic 5 | Expressing Logarithms in Terms of Given Values

Writing a logarithm in terms of other given logarithms • Questions 33–49

Q **33**

Let $a > 1$ and $y > 0$. If $\log_8 a = x + 1$ and $\log_a y = \frac{1}{3}$, then $2^x =$

A) y

B) $y - 2$

C) $\frac{y}{2}$

D) $2y$

E) y^2

Q **34**

If $\log_3(5) = y$, then $\log_5(9) =$

A) $\frac{2}{y}$

B) $\frac{3}{y}$

C) $3y$

D) $2y$

E) y^2

Q
35

If $\frac{1}{x}\log_2(e) = y$, then $\ln 8 =$

A) $\frac{3}{xy}$

B) $\frac{3y}{x}$

C) $\frac{3x}{y}$

D) $\frac{x}{3y}$

E) $3xy$

Q
36

Let $x > 1$. If $\log_{0.5}(x-1) = a$, then $\log_8(2x^2 - 4x + 2) =$

A) $\frac{1+2a}{3}$

B) $\frac{2a-1}{3}$

C) $1 - 2a$

D) $\frac{1-2a}{3}$

E) $\frac{1-2a}{6}$

Q
37

If $\log_3(5) = a$ and $\log_3(2) = b$, then $\log_{\sqrt{2}}(30) =$

A) $\frac{2+2a+2b}{b}$

B) $\frac{2a+2b+2b^2}{b}$

C) $\frac{\sqrt{1+a+b}}{b}$

D) $\frac{2a}{b^2}$

E) $2 + 2a + 2b$

Q
38

If $\log 5 = a$ and $\log 3 = b$, then $\log_3(45) =$

A) $\frac{a+2b}{a}$

B) $\frac{2a+2b}{b}$

C) $\frac{a+2b}{b}$

D) $\frac{2a+b}{b}$

E) $\frac{a+b}{2b}$

Q
39

If $\log_6 3 = a$, then $\log_2 108 =$

A) $\frac{a+2}{1-a}$

B) $\frac{a-2}{1-a}$

C) $\frac{a+3}{1-a}$

D) $\frac{a-3}{1-a}$

E) $\frac{a-2}{1+a}$

Q
40

If $\log_6 2 = x$, then $\frac{1}{2}\log_2 144 =$

A) $\frac{x+1}{x}$

B) $x + 1$

C) $\frac{x}{x-1}$

D) $\frac{1}{x}$

E) $\sqrt{x}$

Q
41

If $\log 2 = t$, then $\log 800 - \log\left(\frac{1}{25}\right) =$

A) $t + 4$

B) $t + 2$

C) $5t + 4$

D) $5t + 2$

E) $2t + 3$

Q
42

If $\log 2 = x$, then $\log 1600 + \log\frac{1}{5}$ is equal to

A) $5x - 1$

B) $3x + 3$

C) $3x$

D) $5x + 1$

E) $5x - 2$

Q
43

If $\log 2 = a$ and $\log 3 = b$, then $\frac{\log_2 \frac{9}{2}}{\log_2 10}$ in terms of a and b is equal to

A) $2b - a$

B) $\frac{a+b}{2}$

C) $\frac{2}{ab}$

D) $\frac{ab}{2}$

E) $a - 2b$

Q
44

If $\log_3 2 = a$ and $\log_3 5 = b$, then $\log 9$ in terms of a and b is equal to

A) $\frac{2}{a+b}$

B) $\frac{1}{a} + \frac{2}{b}$

C) $\frac{ab}{2}$

D) $\frac{a+b}{2}$

E) $\frac{2}{ab}$

Q
45

If $\ln 2 = x$ and $\ln 6 = y$, then $\log_9(12) =$

A) $\frac{x+y}{2y-2x}$

B) $\frac{x+y}{y+2x}$

C) $\frac{x-y}{2y+2x}$

D) $\frac{x+y}{x-y}$

E) $\frac{x-y}{x+y}$

Q
46

If $\log 2 = x$ and $\log 6 = y$, then $\log 15 =$

A) $y - 2x + 1$

B) $2y - x + 1$

C) $2y - x + 2$

D) $\frac{y-x}{x}$

E) $\frac{y+x}{x}$

Q
47

If $\ln(4) = x$ and $\ln(5) = y$, then $\log_{4/5}\left(\frac{16e^2}{125}\right)$ can be written in terms of x and y as:

A) $\frac{2x+2-3y}{x-y}$

B) $\frac{2x-2-3y}{x-y}$

C) $\frac{2x-2+3y}{x-y}$

D) $\frac{2x+2-3y}{x+y}$

E) $\frac{2x-2-3y}{x+y}$

Q
48

If $\log 3 = t$ and $\log 2 = x$, then $2 + \log(0.09) + \log 5 =$

A) $2t - x + 1$

B) $3t + x$

C) $3t - x$

D) $2t + 1$

E) $2t + x + 1$

Q
49

If $\log 0.4 = x$, then $\log_2 20 =$

A) $\frac{x+3}{x+1}$

B) $x + 1$

C) $\frac{x+1}{2}$

D) $x - 1$

E) $\frac{x+1}{x-1}$

Answer Key

"Original" shows the question number in the source worksheet.

Q	Original	Ans.	Value
1	1	**A**	$\frac{\log_{2a} 16}{\log_{2a} 4} = 2$ for any real number $a > 0$ with $a \neq \frac{1}{2}$.
2	2	**A**	$\ln\ x > 0$ if $x > 1$.
3	15	**A**	$\frac{\ln\ x}{\ln\ y} = -\frac{\log_x x}{\log_x \frac{1}{y}}$
4	16	**A**	$\log_b \sqrt{x} = \frac{\ln\ x}{2\ln\ b}$
5	41	**A**	$y = \log_{(a-1)} x$ is defined if $a > 1$, $a \neq 2$ and $x > 0$.
6	46	**A**	$\log\left(x + \sqrt{x^2 - 1}\right) = -\log\left(x - \sqrt{x^2 - 1}\right)$ for any $x \geq 1$.
7	4	**B**	2
8	5	**A**	4
9	12	**A**	$-\frac{4}{5}$
10	13	**A**	$-\frac{8}{3}$
11	14	**A**	11
12	24	**A**	$\frac{13}{6}$
13	26	**A**	-1
14	31	**B**	-1
15	35	**A**	0
16	39	**A**	29
17	45	**A**	64
18	47	**A**	-5
19	11	**E**	$\log_5 \frac{xy^2}{z^3}$
20	22	**A**	$\frac{1}{3} + 4\frac{v}{u}$
21	25	**A**	$\log_2\left(\frac{1}{2x^{7/12}}\right)$
22	28	**A**	$\ln\left(\frac{5ex^2}{\sqrt{x+1}}\right)$
23	29	**B**	$\log_2 \frac{y^8}{2(wx)^2}$
24	34	**A**	$\log \frac{y\sqrt{y}}{x}$
25	38	**A**	$\log_7 \frac{5^{5/6}}{m^{1/3}}$
26	49	**A**	$\frac{5}{2} + 4\log_x 2$
27	6	**A**	$4e^{-9}$
28	7	**A**	2
29	21	**A**	10

Q	Original	Ans.	Value
30	42	**A**	a^4
31	44	**A**	4
32	48	**A**	$\frac{1}{4}$
33	3	**C**	$\frac{y}{2}$
34	8	**A**	$\frac{2}{y}$
35	9	**A**	$\frac{3}{xy}$
36	10	**D**	$\frac{1-2a}{3}$
37	17	**A**	$\frac{2+2a+2b}{b}$
38	18	**C**	$\frac{a+2b}{b}$
39	19	**A**	$\frac{a+2}{1-a}$
40	20	**A**	$\frac{x+1}{x}$
41	23	**A**	$t+4$
42	27	**D**	$5x+1$
43	30	**A**	$2b-a$
44	32	**A**	$\frac{2}{a+b}$
45	33	**A**	$\frac{x+y}{2y-2x}$
46	36	**A**	$y-2x+1$
47	37	**A**	$\frac{2x+2-3y}{x-y}$
48	40	**A**	$2t-x+1$
49	43	**A**	$\frac{x+3}{x+1}$