CHAPTER 2.8

One-to-One Functions and Their Inverses

Multiple-Choice Old Exam Questions

26 questions • 4 topics • options A–E • two questions per page

Topic 1 | Finding Inverse Functions

Finding the formula or constants of the inverse function of rational, radical and quadratic functions • Questions 1–8

Q **1**

If $f^{-1}(x) = \frac{ax+b}{cx+d}$ is the inverse function of $f(x) = \frac{4x+3}{1-x}$, then $a + b + c + d =$

A) 3

B) 9

C) 5

D) 2

E) 7

Q **2**

If $f(x) = 1 - \sqrt{x+2}$ and $f^{-1}(x) = x^2 + ax + b, x \leq 1$, then $a + b =$

A) -3

B) 2

C) 4

D) -4

E) 0

Q
3

If $f^{-1}(x) = \sqrt{x+a} + b$ is the inverse function of $f(x) = x^2 - 2x, x \geq 1$, then $a + b =$

A) 2

B) -2

C) 0

D) 1

E) -1

Q
4

If $f(x) = x^2 - 4x, x \geq 2$, then the inverse of f is

A) $f^{-1}(x) = 2 - \sqrt{x+4},\ x \geq -4$

B) $f^{-1}(x) = 2 + \sqrt{x+4},\ x \geq -4$

C) $f^{-1}(x) = 4 + \sqrt{x+2},\ x \geq -2$

D) $f^{-1}(x) = 4 - \sqrt{x+2},\ x \geq -2$

E) $f^{-1}(x) = 2 + \sqrt{x-4},\ x \geq 4$

Q
5

If $f(x) = \frac{1}{x-1}$, $x \neq 1$, and $f^{-1}(x) = \frac{ax+b}{x}$, then $a + b =$

A) 1

B) -1

C) 2

D) 0

E) -2

Q
6

If $f(x) = \frac{x-3}{x+4}$, $x \neq -4$, and $f^{-1}(x) = \frac{ax+b}{cx+1}$, then $a + b + c =$

A) 6

B) -6

C) 0

D) -8

E) 8

Q
7

If $f(x) = -\sqrt{x^2 - 16}$ for $x \geq 4$, then the inverse function is

A) $f^{-1}(x) = \sqrt{x^2 + 16}$, for $x \leq 0$

B) $f^{-1}(x) = \sqrt{x^2 - 16}$, for $x \geq 0$

C) $f^{-1}(x) = \sqrt{x - 4}$, for $x \geq 4$

D) $f^{-1}(x) = \sqrt{x + 4}$, for $x \geq -4$

E) $f^{-1}(x) = \sqrt{x^2 + 16}$, for $-4 \leq x \leq 4$

Q
8

If $f(x) = \frac{2x+1}{x-1}$, $x \neq 1$, then $f^{-1}(x)$ is equal to

A) $\frac{x+1}{x-2}$, $x \neq 2$

B) $\frac{x+1}{x+2}$, $x \neq -2$

C) $\frac{x-1}{x-2}$, $x \neq 2$

D) $\frac{x-1}{x+2}$, $x \neq -2$

E) $\frac{x-1}{2x+1}$, $x \neq -\frac{1}{2}$

Topic 2 | Values, Compositions and Parameters

Evaluating inverses and compositions, and finding unknown constants • Questions 9–16

Q **9**

Which one of the following statements is FALSE about the function $f(x) = 2 + \sqrt{x}$?

A) The domain of f is $[0, \infty)$.

B) The range of f is $[2, \infty)$.

C) $f^{-1}(1)$ is undefined.

D) $(f^{-1} \circ f)(-4) = -4$.

E) The function f is one-to-one.

Q **10**

If $f(x) = ax + b$, $g(x) = 3x + 2$, and $g(x) = 2f^{-1}(x)$, then $a \cdot b$ is equal to

A) $-\frac{4}{9}$

B) $\frac{4}{9}$

C) 3

D) 1

E) -3

Q
11

If $f(x) = -\sqrt{x+2} + k$ and $f^{-1}(2) = 7$, then $f^{-1}(3) + (f^{-1} \circ f)(2) =$

A) 4

B) $\frac{5}{2}$

C) $\frac{11}{5}$

D) 0

E) 8

Q
12

If $f(x) = x^2 + 2, x < 0$, then $(f^{-1} \circ f)(-1) + f^{-1}(6) =$

A) -3

B) 3

C) $\sqrt{5}$

D) $2\sqrt{3}$

E) 9

Q
13

If $h(x) = (g \circ f)(x)$, where $f(x) = \frac{3}{x-3}$ and $g(x) = \frac{2}{x}$, then $h^{-1}(x) =$

A) $\frac{3}{2}x + 3$

B) $\frac{3}{2}x - 3$

C) $\frac{3x+2}{3x}$

D) $\frac{3x-2}{3x}$

E) $\frac{3}{x-3}$

Q
14

If $f^{-1}(x) = \frac{1}{2} - \sqrt{x + \frac{5}{4}}$, then $f\left(-\frac{1}{2}\right)$ is equal to

A) $-\frac{1}{4}$

B) $-\frac{9}{4}$

C) $\frac{9}{4}$

D) $\frac{1-\sqrt{3}}{2}$

E) $-\frac{5}{4}$

Q
15

If $f(x) = a^{x+b}$, where $f^{-1}(1) = 4$ and $f^{-1}(3) = 5$, then $f(2) =$

A) $\frac{1}{9}$

B) 9

C) 3

D) $\frac{1}{3}$

E) -4

Q
16

If $f^{-1}(x) = \frac{x+a}{x+b}$ is the inverse function of $f(x) = \frac{kx+2}{x-1}$, where $f(2) = -k$, then $a + b =$

A) $\frac{8}{3}$

B) $\frac{4}{3}$

C) -3

D) -1

E) 3

Topic 3 | Domain, Range and Graphs of Inverses

Using the domain and range of a function to describe its inverse • Questions 17–20

Q **17**

If $f(x) = \frac{1}{x+2}$, $x \neq -2$, then the graph of $f^{-1}(x)$ lies below the x-axis over the interval

A) $(-\infty, 0) \cup \left(\frac{1}{2}, \infty\right)$

B) $(-\infty, 0) \cup (0, \infty)$

C) $(-\infty, -2) \cup (-2, \infty)$

D) $(-\infty, -2) \cup (0, \infty)$

E) $(-\infty, 0) \cup (2, \infty)$

Q **18**

If $f^{-1}(x) = -\sqrt{x+9}$, $x \geq -9$, then the graph of f lies below the x-axis on the interval

A) $[0, \infty)$

B) $(-9, 0]$

C) $(-3, 0]$

D) $(3, \infty)$

E) $(-\infty, 0]$

Q
19

Given the function $f(x) = -\sqrt{16 - x^2}$, $0 \leq x \leq 4$, the domain of $f^{-1}(x)$ is

A) $[-4, 0]$

B) $[-4, 4]$

C) $[0, 4]$

D) $[4, \infty)$

E) $(-\infty, -4]$

Q
20

If $f(x) = -|x - 3| + 2, x \leq 3$, then the domain of the inverse function f^{-1} is

A) $(-\infty, 2]$

B) $[2, \infty)$

C) $[3, \infty)$

D) $(-\infty, \infty)$

E) $(-\infty, 3]$

Topic 4 | One-to-One Functions and True/False Statements

Testing whether a function is one-to-one and judging statements about inverse functions • Questions 21–26

Q **21**

Which one of the following functions is NOT one-to-one?

A) $f(x) = |x - 1| + 2$

B) $f(x) = x^3 - 6$

C) $f(x) = x^2 - 4,\ 0 \leq x < \infty$

D) $f(x) = 3x - 5$

E) $f(x) = -\frac{2}{x+3}$

Q **22**

Which one of the following functions is NOT a one-to-one function?

A) $f(x) = \sqrt{(x-2)^2},\ x \geq 0$

B) $f(x) = 2 - \sqrt{2x - 1}$

C) $f(x) = \frac{1}{x-1} + 3$

D) $f(x) = x^2 - 2x + 1,\ x \leq 1$

E) $f(x) = (x-1)^3$

Q
23

Which one of the following statements is FALSE?

A) The function $f(x) = x^2 + 1, x < 1$, is a one-to-one function.

B) If $f(x) = 2^x$, then $f^{-1}(x) = \log_2 x$.

C) If $f(x) = x$, then $f^{-1}(x) = x$.

D) $f(x) = 5$ is NOT a one-to-one function.

E) If f is a one-to-one function, then f^{-1} exists.

Q
24

Which one of the following statements is FALSE?

A) If $f = \{(-1, 2), (2, 1), (5, -1)\}$, then $f^{-1} = \{(2, -1), (1, 2), (-1, 2)\}$.

B) If f is a one-to-one function, then $g(x) = f(x) + 5$ is a one-to-one function.

C) If f is a one-to-one function, then f^{-1} is a one-to-one function.

D) If $f(x) = x^2$ for all $x < 0$, then the range of f^{-1} is $(-\infty, 0)$.

E) If $f(x) = x + 1$, then the domain of f^{-1} is $(-\infty, \infty)$.

Q
25

Which one of the following statements is FALSE about inverse functions?

A) If $f(2) = -5$, then $f(f^{-1}(-5)) = 2$.

B) For a function to have an inverse, it must be a one-to-one function.

C) If the point (a, b) lies on the graph of f, then (b, a) lies on the graph of f^{-1}.

D) The domain of f is equal to the range of f^{-1}.

E) The graphs of f and f^{-1} are symmetric with respect to the line $y = x$.

Q
26

Which one of the following statements is FALSE?

A) If $f(x) = x^2$, then $f^{-1}(x) = \sqrt{x}$.

B) The function $f(x) = 3$, defined over the set of real numbers, is not one-to-one.

C) The range of the function f is equal to the domain of f^{-1}.

D) An increasing function on its entire domain is one-to-one.

E) If the point (a, b) lies on the graph of f, then the point (b, a) lies on the graph of f^{-1}.

Answer Key

"Original" shows the question number in the source worksheet.

Q	Original	Ans.	Value
1	1	**A**	3
2	2	**A**	-3
3	3	**A**	2
4	5	**B**	$f^{-1}(x) = 2 + \sqrt{x+4},\ x \geq -4$
5	7	**C**	2
6	14	**A**	6
7	16	**A**	$f^{-1}(x) = \sqrt{x^2+16},$ for $x \leq 0$
8	18	**A**	$\frac{x+1}{x-2},\ x \neq 2$
9	6	**D**	$(f^{-1} \circ f)(-4) = -4.$
10	8	**A**	$-\frac{4}{9}$
11	9	**A**	4
12	10	**A**	-3
13	11	**A**	$\frac{3}{2}x + 3$
14	21	**A**	$-\frac{1}{4}$
15	25	**A**	$\frac{1}{9}$
16	26	**A**	$\frac{8}{3}$
17	15	**A**	$(-\infty, 0) \cup \left(\frac{1}{2}, \infty\right)$
18	20	**C**	$(-3, 0]$
19	23	**A**	$[-4, 0]$
20	24	**A**	$(-\infty, 2]$
21	19	**A**	$f(x) = \lvert x - 1 \rvert + 2$
22	22	**A**	$f(x) = \sqrt{(x-2)^2},\ x \geq 0$
23	13	**A**	The function $f(x) = x^2 + 1, x < 1$, is a one-to-one function.
24	4	**A**	If $f = \{(-1, 2), (2, 1), (5, -1)\}$, then $f^{-1} = \{(2, -1), (1, 2), (-1, 2)\}$.
25	12	**A**	If $f(2) = -5$, then $f(f^{-1}(-5)) = 2$.
26	17	**A**	If $f(x) = x^2$, then $f^{-1}(x) = \sqrt{x}$.