CHAPTER 1.8

Absolute Value Equations and Inequalities

Multiple-Choice Old Exam Questions

37 questions • 5 topics • options A–E • two questions per page

Topic 1 | Basic Absolute Value Equations

Isolating the absolute value, or setting two absolute values equal • Questions 1–8

Q **1**

The sum of all the solutions of $-\frac{3}{2}|x-7|-\frac{5}{2}=-7$ is equal to

A) 14

B) 12

C) -14

D) 6

E) -6

Q **2**

The sum of all the solutions of the equation $\frac{5}{2}-4|3x-6|=-\frac{19}{3}$ is

A) 4

B) 2

C) 8

D) -4

E) 6

Q
3

The solution set of the equation $|-4-3x| = |2-3x|$ contains

A) only one element

B) no element

C) exactly two elements

D) exactly three elements

E) infinitely many elements

Q
4

The solution set of the equation $|5x-1| = |2x+3|$ contains

A) one positive and one negative rational number

B) two positive rational numbers

C) two negative rational numbers

D) one positive and one negative irrational number

E) no real solution

Q
5

The sum of all the solutions of the equation $|x-1| = |3x+2|$ is

A) $-\frac{7}{4}$

B) 1

C) 0

D) $-\frac{1}{4}$

E) $-\frac{3}{2}$

Q
6

The sum of the solutions of the equation $\left|\frac{6x+1}{x-1}\right| = 3$ is equal to

A) $-\frac{10}{9}$

B) $\frac{10}{9}$

C) $-\frac{4}{3}$

D) $\frac{2}{9}$

E) $\frac{14}{9}$

Q
7

The solution set of $\big||x+1|-2\big|=5$ contains

A) two solutions

B) one solution

C) three solutions

D) four solutions

E) five solutions

Q
8

The solution set of the equation $\frac{2|x+2|}{3}-\frac{1}{2}=\frac{4x+5}{6}$ is

A) $[-2,\infty)$

B) $(-\infty,-2]$

C) $(-2,\infty)$

D) $(-\infty,\infty)$

E) $\{-2\}$

Topic 2 | Quadratic-Type and Higher-Degree Equations

Substituting u for an absolute value, and counting or summing the solutions • Questions 9–16

Q **9**

The sum of all the solutions of the equation $|x-2|^2 - |2-x| = 6$ is

A) 4

B) 8

C) -1

D) -8

E) 1

Q **10**

The sum of the solutions of the equation $\frac{|x-1|+2}{1+|x-1|} - \frac{3}{2} = 0$ is equal to

A) 2

B) 1

C) 0

D) -1

E) -5

Q
11

The sum of all the solutions of $|x-6|^2-3|x-6|-4=0$ is equal to

A) 12

B) 10

C) 25

D) 20

E) 15

Q
12

The sum of all the solutions of the equation $|x-1|^2-3|x-1|-4=0$ is

A) 2

B) 5

C) -2

D) -3

E) 3

Q
13

The sum of the solutions of $|(x+2)^2| = |x+2|$ is equal to

A) -6

B) -4

C) -3

D) 0

E) 6

Q
14

The number of solutions of the equation $|x^2 - 2| = |x|$ is

A) 4

B) 1

C) 2

D) 5

E) 3

Q
15

The number of solutions of the equation $|2x-1|^3-5|2x-1|^2+|8x-4|=0$ is

A) 5

B) 3

C) 4

D) 0

E) 6

Q
16

The number of solutions of the equation $|x+3|^2+|x+3|=0$ is equal to

A) 1

B) 0

C) 2

D) 3

E) 4

Topic 3 | Absolute Value Inequalities

Rewriting as compound inequalities and writing the answer in interval notation • Questions 17–24

Q **17**

The solution set of the inequality $8 + 3\left|x - \frac{1}{2}\right| \geq 2$ is

A) $(-\infty, \infty)$

B) $\left(-\infty, -\frac{3}{2}\right] \cup \left[\frac{5}{2}, \infty\right)$

C) $\left[-\frac{3}{2}, \frac{5}{2}\right]$

D) $(-\infty, 0) \cup (0, \infty)$

E) $\emptyset$

Q **18**

The solution set of the inequality $|x - 5| \geq -3$ is

A) $(-\infty, \infty)$

B) $(-\infty, -3]$

C) $[-3, \infty)$

D) $(5, \infty)$

E) $\emptyset$

Q
19

The solution set of the inequality $4 - 2\left|x - \frac{1}{2}\right| < 5$ is

A) $(-\infty, \infty)$

B) $\left(-\infty, \frac{1}{2}\right)$

C) $[0, 1]$

D) $\emptyset$

E) $\left(-\frac{1}{2}, \infty\right)$

Q
20

The solution set of the absolute value inequality $|x|^2 + |x| \geq 2$ is

A) $(-\infty, -1] \cup [1, \infty)$

B) $(-\infty, -2] \cup [1, \infty)$

C) $(-\infty, -2] \cup [2, \infty)$

D) $(-\infty, -1]$

E) $(-\infty, -2]$

Q
21

If $(-\infty, m) \cup (n, \infty)$ is the solution set of the inequality $\left|\frac{5}{3} - \frac{1}{2}x\right| + \frac{1}{3} > \frac{4}{3}$, then $n - m =$

A) 4

B) 15

C) $\frac{20}{3}$

D) $\frac{11}{3}$

E) -4

Q
22

The solution set, in interval notation, of the inequality $\left|\frac{1}{2} - 2x\right| \geq \frac{1}{2}$ is

A) $(-\infty, 0] \cup [1/2, \infty)$

B) $(-\infty, 0] \cup [1, \infty)$

C) $(-\infty, 1/4] \cup [1/2, \infty)$

D) $(-\infty, 0] \cup [1/4, \infty)$

E) $(-\infty, 1/2) \cup [1, \infty)$

Q
23

The solution set of the inequality $\left|\frac{1}{2}x + \frac{2}{3}\right| > 3$, in interval notation, is

A) $\left(-\infty, -\frac{22}{3}\right) \cup \left(\frac{14}{3}, \infty\right)$

B) $\left(-\frac{22}{3}, \frac{14}{3}\right)$

C) $\left(-\infty, -\frac{14}{3}\right) \cup \left(\frac{22}{3}, \infty\right)$

D) $\left(-\frac{14}{3}, \frac{22}{3}\right)$

E) $\left(-\infty, -\frac{22}{3}\right] \cup \left[\frac{14}{3}, \infty\right)$

Q
24

The solution set, in interval notation, of the inequality $\left|\frac{2}{3}x - 1\right| - 2 > \frac{1}{3}$ is

A) $(-\infty, -2) \cup (5, \infty)$

B) $(-2, 5)$

C) $(-\infty, -2] \cup [5, \infty)$

D) $(-\infty, -5) \cup (2, \infty)$

E) $(-\infty, \infty)$

Topic 4 | Compound Inequalities, Intersections and Parameters

Rational absolute value inequalities, intersections of solution sets, and unknown constants • Questions 25–34

Q **25**

The solution set of the inequality $\left|\frac{3}{x-6}\right| > 1$ is

A) $(-\infty, 3) \cup (9, \infty)$

B) $(3, 6) \cup (6, 9)$

C) $(3, 9)$

D) $(-\infty, 3] \cup [9, \infty)$

E) $(6, 9)$

Q **26**

The solution set of the inequality $\left|\frac{2x+5}{x}\right| \leq 1$, in interval notation, is

A) $\left[-5, -\frac{5}{3}\right]$

B) $[-5, 0)$

C) $(-\infty, -5] \cup \left[-\frac{5}{3}, \infty\right)$

D) $\left[-5, -\frac{5}{3}\right)$

E) $\left[-\frac{5}{3}, 0\right)$

Q
27

The solution set, in interval notation, of the inequality $2 < |x - 1| < 3$ is equal to

A) $(-2, -1) \cup (3, 4)$

B) $(-\infty, -2) \cup (4, \infty)$

C) $[-2, -1] \cup [3, 4]$

D) $(-3, -2) \cup (2, 3)$

E) $(-2, 4)$

Q
28

The solution set of the compound inequality $-3|x| + 6 < 12$ and $8 - |2x - 1| \geq 6$ is equal to

A) $[-1/2, 3/2]$

B) $(-\infty, \infty)$

C) $[-1/2, 0) \cup (0, 3/2]$

D) $(-\infty, 1/2] \cup [3/2, \infty)$

E) the empty set $\emptyset$

Q
29

If $x \leq -\frac{1}{2}$, then the solution set of the inequality $|2x-1| \geq |x^2-2x|$ is

A) $\left[-1, -\frac{1}{2}\right]$

B) $\emptyset$

C) $[-2, -1]$

D) $(-\infty, -1]$

E) $\left(-\infty, -\frac{1}{2}\right]$

Q
30

If A is the solution set of $|3-2x| \leq 5$ and B is the solution set of $|x-2| > 1$, then $A \cap B =$

A) $[-1, 1) \cup (3, 4]$

B) $[-1, 4]$

C) $(1, 3)$

D) $[-1, 1] \cup [3, 4]$

E) $(-\infty, 1) \cup (3, \infty)$

Q
31

Let A be the solution set of the inequality $4|x-3| > 12$ and let B be the solution set of the inequality $|5-3x| \leq 7$. Then $A \cap B =$

A) $\left[-\frac{2}{3}, 0\right)$

B) $\left[-\frac{2}{3}, 4\right]$

C) $\left(-\frac{2}{3}, 0\right]$

D) $(6, \infty)$

E) $(-\infty, 0) \cup (6, \infty)$

Q
32

If $|3-2x| \leq 5$ is equivalent to $m \leq 5x+2 \leq n$, then

A) $m = -3$ and $n = 22$

B) $m = -3$ and $n = 12$

C) $m = -8$ and $n = 22$

D) $m = -1$ and $n = 4$

E) $m = 2$ and $n = 22$

Q
33

If the solution set of $|x - a| < 1 - 2a$ is $-10 < x < 4$, then $a =$

A) -3

B) -5

C) -6

D) -4

E) -7

Q
34

If the solution of $\left|x + \frac{6}{k+1}\right| \leq \frac{3}{k+1}$ is $[-3, -1]$, then $k =$

A) $\{2\}$

B) $\{2, -3\}$

C) $\{-1/2, 2\}$

D) $\{-2, -4\}$

E) $\emptyset$

Topic 5 | Special Cases and Reasoning

Equations with no solution, products with absolute values, and simplifying radicals • Questions 35–37

Q **35**

The solution set of the equation $\frac{1}{x+6} - \frac{|4x-7|}{x^2+5x-6} = \frac{1}{x-1}$ consists of

A) no real solution

B) exactly one real solution

C) exactly two real solutions

D) exactly three real solutions

E) all real numbers except -6 and 1

Q **36**

If $x^2|y| + y|x^2| = 0$, then

A) $x = 0$ or $y \leq 0$

B) $x = 0$ or $y > 0$

C) $x < 0$ or $y \leq 0$

D) $x > 0$ or $y \geq 0$

E) $x < 0$ or $y > 0$

Q
37

If $x < 3$, then $\sqrt{x^2 - 6x + 9} - x - 2 =$

A) $2x - 1$

B) $-2x + 1$

C) -5

D) -1

E) 1

Answer Key

"Original" shows the question number in the source worksheet.

Q	Original	Ans.	Value	Q	Original	Ans.	Value
1	1	**A**	14	**20**	36	**A**	$(-\infty, -1] \cup [1, \infty)$
2	13	**A**	4	**21**	3	**A**	4
3	21	**A**	only one element	**22**	10	**A**	$(-\infty, 0] \cup [1/2, \infty)$
4	23	**A**	one positive and one negative rational number	**23**	22	**A**	$\left(-\infty, -\frac{22}{3}\right) \cup \left(\frac{14}{3}, \infty\right)$
5	28	**A**	$-\frac{7}{4}$	**24**	19	**A**	$(-\infty, -2) \cup (5, \infty)$
6	20	**A**	$-\frac{10}{9}$	**25**	30	**B**	$(3, 6) \cup (6, 9)$
7	7	**A**	two solutions	**26**	24	**A**	$\left[-5, -\frac{5}{3}\right]$
8	15	**A**	$[-2, \infty)$	**27**	16	**A**	$(-2, -1) \cup (3, 4)$
9	2	**A**	4	**28**	11	**A**	$[-1/2, 3/2]$
10	8	**A**	2	**29**	5	**A**	$\left[-1, -\frac{1}{2}\right]$
11	26	**A**	12	**30**	17	**A**	$[-1, 1) \cup (3, 4]$
12	29	**A**	2	**31**	18	**A**	$\left[-\frac{2}{3}, 0\right)$
13	25	**A**	-6	**32**	14	**A**	$m = -3$ and $n = 22$
14	27	**A**	4	**33**	4	**A**	-3
15	9	**A**	5	**34**	12	**A**	$\{2\}$
16	34	**A**	1	**35**	32	**A**	no real solution
17	6	**A**	$(-\infty, \infty)$	**36**	35	**A**	$x = 0$ or $y \leq 0$
18	31	**A**	$(-\infty, \infty)$	**37**	37	**B**	$-2x + 1$
19	33	**A**	$(-\infty, \infty)$				