King Fahd University of Petroleum and Minerals
Prep-Year Math Program

Prep-Year Math I
FINAL EXAM
Term 091
Tuesday, February 02, 2010

CODE 002		CODE 002

Net Time Allowed: **180 minutes**

Student's Name: ....

ID #: .... **Section #:** ....

Check that the exam paper has **36 questions**

Important Instructions:

1. All types of ***calculators, pagers, or mobiles*** are ***not allowed*** to be with you during the examination.
2. Write ***your name***, ***ID number*** and Mathematics ***section number*** on the examination paper and in the upper left corner of the answer sheet.
3. When bubbling your ID number and Math section number, be sure that bubbles match with the number that you write
4. The test code number is already typed and bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
5. When bubbling, make sure that the bubbled space is fully covered.
6. When erasing a bubble, make sure that you do not leave any trace of penciling.

Q1.
The **vertex**, **axis** of symmetry of the parabola having equation $f(x) = -2(x-1)(x+3)$ and the **range** of f are:

A) vertex: $(-1, 0)$; axis: $x=-1$; range: $(-\infty, 0]$
B) vertex: $(0, 6)$; axis: $x=0$; range: $[6, \infty)$
C) vertex: $(1, -3)$; axis: $x=1$; range: $(-\infty, -3]$
D) vertex: $(8, -1)$; axis: $x=8$; range: $[-1, \infty)$
E) vertex: $(-1, 8)$; axis: $x=-1$; range: $(-\infty, 8]$

$y = -2(x-1)(x+3) = -2\left[x^2+3x-x-3\right]$

$y = -2\left[(x+1)^2-(1)^2-3\right]$

$y = -2(x+1)^2+2+6$

$= -2(x+1)^2+8$

$\frac{y-8}{-2} = (x+1)^2$

$-\frac{1}{2}(y-8) = (x+1)^2 \Longrightarrow$ Vertex $= (-1, 8)$; axis : $x = -1$;
Range $= (-\infty, 8)$

Q2.
Which one of the following statements is TRUE about the function $f(x) = x|x|$?

A) The graph of f is the same as the graph of $g(x) = x^2$
B) f is decreasing on its domain
C) The graph of f is symmetric about the origin
D) The range of f is $[0, \infty)$
E) f is an even function

(E) Replace x by $-x$.
$f(-x) = -x\,|-x| = -x\,|x| = -f(x)$ not even

(C) Replace x by $-x$ and y by $-y$.
$\Rightarrow$ symmetric about the origin.

Q3.

The solution set of the equation $\dfrac{3}{x+4}+\dfrac{4}{x+3}=\dfrac{4}{x^2+7x+12}$ contains:

A) two positive real numbers
B) no real numbers
C) two negative real numbers
D) one positive real number only
E) one negative real number only

$$\frac{3(x+3)+4(x+4)}{x^2+7x+12}=\frac{4}{x^2+7x+12}$$

$$\Longrightarrow 3(x+3)+4(x+4)=4$$

$$3x+9+4x+16=4$$

$$7x=4-9-16$$

$$7x=-21$$

$$x=-3$$

Not a solution because $x+3$ is denominator.

Q4.

The **number** of **rational zeros** of the polynomial $p(x)=2x^4-4x^3+3x^2+9x$ is:

Use Descarte Rule.

A) 4
B) 1
C) 0
D) 2
E) 3

$p(x): 2x^4-4x^3+3x^2+9x$

$\therefore$ 2 or zero positive roots.

$p(-x)= 2x^4+4x^3+3x^2-9x$

1 negative zeros.

$$\left.\begin{array}{l} x=0 \text{ is root} \\ x=-1 \text{ is root} \end{array}\right\} \Longrightarrow 2 \text{ rational zeros.}$$

Q5.

If $f(x)=\sqrt{6+x}$, $x\geq -6$, and f^{-1} is the inverse function of f then $\left(f\circ f^{-1}\right)(4)+f^{-1}(5)=$

A) 29
B) 23
C) 15
D) 25
E) 35

$f(x)=\sqrt{6+x}$

$y=\sqrt{6+x}$

~~Replace~~ Swap x ~~by~~, y

$x=\sqrt{6+y}$

$x^2=6+y$

$y=x^2-6$

$f^{-1}(x)=x^2-6.$

$f^{-1}(5)=25-6=19.$

$(f\circ f^{-1})(4)=4.$

$(f\circ f^{-1})(4)+f^{-1}(5)=$

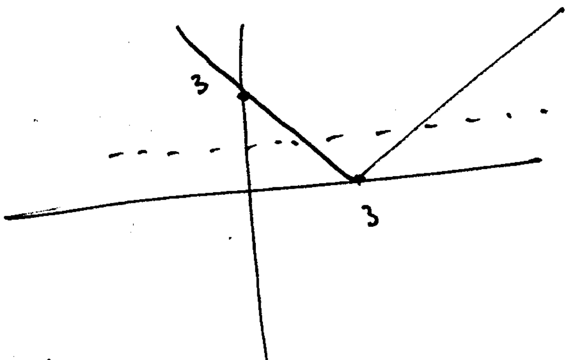
$4+19=23$

Q6.

Which one of the following functions is **NOT** a one-to-one function?

A) $f(x)=\sqrt{(x-3)^2}$
B) $f(x)=1-\sqrt{1-x}$
C) $f(x)=\dfrac{x+1}{5}$
D) $f(x)=(x-1)^3$
E) $f(x)=1+\dfrac{1}{x}$

$\sqrt{(x-3)^2}=|x-3|$



Horizontal line test. fails.

Q7.

If the denominator is rationalized, then the expression $\frac{\sqrt{5}+2\sqrt{2}}{3\sqrt{2}+2\sqrt{5}}$ simplifies to:

A) $\frac{\sqrt{10}}{2}-1$

B) $\frac{-\sqrt{10}}{2}-1$

C) $\frac{\sqrt{10}}{2}-\frac{1}{2}$

D) $\frac{-\sqrt{10}}{4}+\frac{1}{2}$

E) $\frac{\sqrt{10}}{2}+1$

$$\frac{\sqrt{5}+2\sqrt{2}}{3\sqrt{2}+2\sqrt{5}} = \frac{(\sqrt{5}+2\sqrt{2})(3\sqrt{2}-2\sqrt{5})}{(3\sqrt{2}+2\sqrt{5})(3\sqrt{2}-2\sqrt{5})}$$

$$= \frac{3\sqrt{10}-10+12-4\sqrt{10}}{18-20}$$

$$= \frac{2-\sqrt{10}}{-2} = \frac{\sqrt{10}-2}{2} = \frac{\sqrt{10}}{2}-1$$

Q8.

If $f(x)=2x^2+5$, $g(x)=2x+m$ and the graph of the function $(f\circ g)(x)$ has y-intercept 23, then $m=$

A) -5

B) $\pm\sqrt{7}$

C) 5

D) ± 7

E) ± 3

$f(x) = 2x^2+5$

$g(x) = 2x+m$

$f\circ g(x) = 2(2x+m)^2+5$

$= 2(4x^2+4mx+m^2)+5$

$= 8x^2+8mx+2m^2+5$

y-intercept if $x=0$

$\Rightarrow 2m^2+5=23$

$2m^2=18$

$m^2=9 \Rightarrow m=\pm 3.$

Q9.

An equation of the line passing through the point $(3, 5)$ and perpendicular to the line $2x+5y=4$, is given by:

A) $5x-2y-5=0$
B) $5x-2y+25=0$
C) $2x+5y-31=0$
D) $5x+2y+15=0$
E) $2x+5y-5=0$

Slope of $2x+5y=4$. is $-\frac{2}{5}$

as $5y = 4-2x$.

$y = \frac{4}{5} - \frac{2x}{5}$

Slope of other line is $\frac{5}{2}$.

So we get $y = \frac{5}{2}x + b$

Substituting $(3,5)$ in equation

$5 = \frac{5}{2}(3) + b$

Solve for b $\Rightarrow b = -\frac{5}{2}$.

$\Rightarrow y = \frac{5}{2}x - \frac{5}{2}$ or $2y = 5x-5$ or $5x-2y-5=0$.

Q10.

If the graph of $f(x)=|x|$ is reflected across the x-axis; then translated two units left and three units down, then the equation of the new graph is:

A) $g(x)=-|x+2|-3$
B) $g(x)=|-x+2|-3$
C) $g(x)=|-x-2|+3$
D) $g(x)=-|x-3|+2$
E) $g(x)=-|x-2|+3$

Reflexion $f(x) = -|x|$

translation 2 units left

$f(x) = -|x+2|$

translation 3 units down

$f(x) = -|x+2|-3$

Q11.

Solving the equation $x^2 + xy + y = 1$, where $y \geq 2$, for x in terms of y, one solution is:

A) $x = 2 - y$
B) $x = 2y - 1$
C) $x = 1 - y$
D) $x = |2 - y|$
E) $x = |1 - y|$

$$x = \frac{-y \pm \sqrt{y^2 - 4(1)(y-1)}}{2}$$

$$= \frac{-y \pm \sqrt{y^2 - 4y + 4}}{2}$$

$$= \frac{-y \pm \sqrt{(y-2)^2}}{2}$$

$$= \frac{-y \pm y - 2}{2}$$

$x_1 = \frac{-y - y + 2}{2} = \frac{-2y+2}{2} = 1 - y$ and $x_2 = \frac{-y + y - 2}{2} = -1$

Q12.

If $f(x) = \frac{x-3}{x+4}$ and $f^{-1}(x) = \frac{ax+b}{1+cx}$, then $a + b + c =$

A) 8
B) 6
C) -8
D) 2
E) 0

$$y = \frac{x-3}{x+4}$$

$$x = \frac{y-3}{y+4} \Rightarrow xy + 4x = y - 3$$

$$y(x-1) = -4x - 3$$

$$y = \frac{-4x - 3}{x - 1}$$

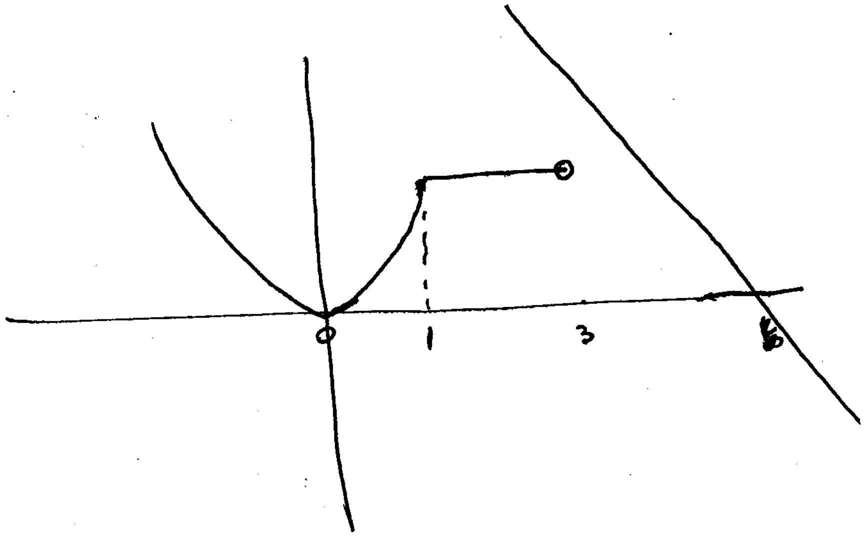
$$= \frac{4x+3}{1-x} \Rightarrow a = 4,\ b = 3,\ c = -1 \Rightarrow a + b + c = 6$$

Q13.

The function $f(x)=\begin{cases} x^2 \text{ if } x \le 1 \\ 1 \text{ if } 1 < x < 3 \\ 6-x \text{ if } x \ge 3 \end{cases}$ is increasing on the interval(s):

[Hint: Graph the function f]

A) $[0, 1]$
B) $(-\infty, 1] \cup [3, \infty)$
C) $(-\infty, 0] \cup (3, \infty)$
D) $(3, \infty)$
E) $(-\infty, 1]$



increasing $[0, 1]$.

Q14.
If x_1 and x_2 are the real solutions of the equation $16x^4 + 12x^2 - 4 = 0$, then $\frac{x_1}{x_2} =$

A) 4
B) -1
C) $\frac{1}{4}$
D) $-\frac{1}{4}$
E) -4

$16x^4 + 12x^2 - 4 = 0$

$4x^4 + 3x^2 - 1 = 0$

$(4x^2 - 1)(x^2 + 1) = 0$

$4x^2 = 1$ or $x^2 = -1$

$x^2 = \frac{1}{4}$

$x = \pm\frac{1}{2}$ or $x = \pm i$

$\frac{\frac{1}{2}}{-\frac{1}{2}} = -1$

Q15.
If $y = mx + b$ is the equation of the oblique asymptote to the graph of the rational function $f(x) = \dfrac{2x^2 + x - 6}{x - 1}$, then $m + b =$

A) 1
B) 2
C) 3
D) 5
E) 4

Using ~~synthetic~~ synthetic division

$$\begin{array}{c|ccc} 1 & 2 & 1 & -6 \\ & & 2 & 3 \\ \hline & 2 & 3 & -3 \end{array}$$

So Quotient is $2x + 3$
So oblique asymptote is $y = 2x + 3$
$m = 2$
$b = 3$

Q16.
The distance between the point $(-4, -4)$ and the center of the circle $x^2 + y^2 - 6x + 10y + 25 = 0$ is equal to:

A) $4\sqrt{2}$
B) $3\sqrt{2}$
C) $2\sqrt{5}$
D) $10\sqrt{5}$
E) $5\sqrt{2}$

Completing the square.

$x^2 + y^2 - 6x + 10y + 25 = 0$

$x^2 - 6x + y^2 + 10y = 25$

$(x-3)^2 - (3)^2 + (y+5)^2 - (5)^2 = 25$

$\Rightarrow$ Centre $= (3, -5)$.

Distance between centre and $(-4, -4)$ is

$\sqrt{(-4-3)^2 + (-4+5)^2} = \sqrt{(-7)^2 + 1^2} = \sqrt{50}$
$= 5\sqrt{2}$

Q17.

The sum of the **real** and **imaginary** parts of the complex number

$z = \dfrac{7+i}{3+4i} + \sqrt{-4}$

is equal to:

A) −2
B) −3
C) 3
D) 0
E) 2

$$\frac{7+i}{3+4i} + \sqrt{-4} = \frac{7+i}{3+4i} + 2i$$

$$= \frac{(7+i)(3-4i)}{9+16} + 2i$$

$$= \frac{21 - 28i + 3i - 4i^2}{25} + 2i$$

$$= \frac{25 - 25i}{25} + 2i$$

$$= 1 - i + 2i = 1 + i.$$

Real part = 1
imaginary part = 1

Q18.

If $y = 3$ is the horizontal asymptote of the function $y = \dfrac{ax+3}{1-2x}$, then the x-intercept of the graph is:

A) $\dfrac{3}{2}$
B) −3
C) $-\dfrac{2}{3}$
D) $\dfrac{1}{2}$
E) −2

Horizontal asymptote $= \dfrac{a}{-2} = 3$

$\Rightarrow a = -6.$

$y = \dfrac{-6x+3}{1-2x}$

x-intercept is given by

$-6x+3=0$

$-6x = -3$

$x = \dfrac{1}{2}.$

Q19.

The domain of the function $g(x)=\frac{1}{x^2-1}\sqrt{1-\frac{1}{x}}$ is:

A) $(-\infty,-1)\cup(-1,0)\cup(1,\infty)$
B) $(-\infty,-1)\cup(1,\infty)$
C) $(-\infty,0)\cup(0,\infty)$
D) $(-1,1)$
E) $(-\infty,-1)\cup(0,\infty)$

$x^2 \neq 1$

$x \neq \pm 1$

and $1-\frac{1}{x} \geq 0$ and $x \neq 0$

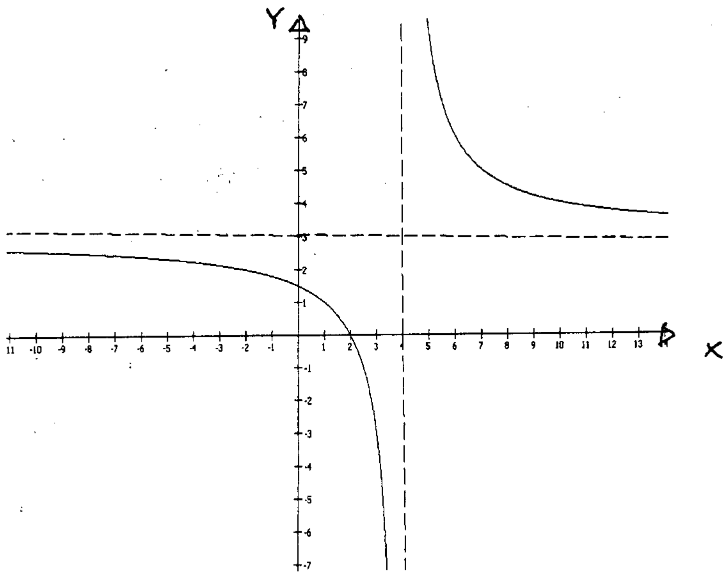
$\frac{1}{x} \leq 1$

$1 \leq x$

Hence $(-\infty,1)\cup(-1,0)\cup(1,\infty)$.

Q20.
Which one of the following functions has the graph given below?

A) $f(x)=\frac{3-x}{4-x}$
B) $f(x)=\frac{2-3x}{4-x}$
C) $f(x)=\frac{x-3}{x-4}$
D) $f(x)=\frac{6-3x}{4-x}$
E) $f(x)=\frac{3x-12}{4x-16}$



---

x- intercept is 2 $\Longrightarrow$ choice E, C, B and A are wrong.

$\Longrightarrow$ $f(x)=\frac{6-3x}{4-x}$.

Also give the correct vertical asymptote and horizontal asymptote.

Q21.

The polynomial $p(x) = 2x^4 - 4x^2 + 4x - 8$ has a real zero between:

A) -1 and 0
B) 3 and 4
C) 0 and 1
D) 2 and 3
E) 1 and 2

Check using intermediate value Theorem.

$p(1) = 2 - 4 + 4 - 8 < 0$

$p(2) = 2(16) - 4(4) + 8 - 8 > 0$

$\therefore$ Root between 1 and 2.

Q22.

The **range** of the function $y = \sqrt{16 - x^2}$ is given by:

A) $-4 \le y \le 4$
B) $-\infty < y < \infty$
C) $0 \le y \le 4$
D) $y \le 4$
E) $y \ge 0$

domain is

$16 - x^2 \geq 0$

$-4 \le x \le 4$

So y is min when $x = 4$ or $x =$

i.e. $y = 0$

y is maxi if $x = 0$

i.e., $y = \sqrt{16} =$

Q23.
Which one of the following statements is **true** about the graph of the polynomial function
$f(x) = x^2(x-3)^3(x+1)$?

A) the graph is increasing in the interval $(-\infty, -1]$
B) the graph lies above the x-axis in the interval $(-1, 3)$
C) the graph crosses the x-axis at three points
D) the graph has y-intercept $= -27$
E) the graph crosses the x-axis at two points

Crosses x-axis at $x = 3$ and $x = -1$ since power of these factors are odd.

Q24.
If the graph of a linear function $y = f(x)$ is passing through the points $(2, 3)$ and $(1, -4)$, then $f(-2) =$

A) −3
B) −5
C) −25
D) 3
E) 25

Slope is $\frac{y_2 - y_1}{x_2 - x_1} = \frac{-4-3}{1-2} = 7$

$\Longrightarrow y = 7x + b$

Substituting at $(2, 3)$

$3 = 7(2) + b$

$3 = 14 + b$

$b = -11$

$\Longrightarrow y = 7x - 11$

$f(-2) = 7(-2) - 11 = -25$

Q25.
Which one of the following statements is TRUE?

A) $\left[5+(-3)\right]^2 = 5^2 + (-3)^2$
B) $3^2 \cdot 3^4 = 3^8$
C) $\sqrt{(-6)^2} = |-6|$
D) $\sqrt{3^2+4^2} = \sqrt{3^2} + \sqrt{4^2}$
E) $(-2)^4 = -2^4$

$\sqrt{(-6)^2} = |-6|$

since the power is even

Q26.
The solution set, in interval notation, of the inequality $x^3+3x^2-4x \le 12$ is:

A) $[-3,-2] \cup [2,\infty)$
B) $(-\infty, 2]$
C) $(-\infty,-2] \cup [2,3]$
D) $(-\infty,-2]$
E) $(-\infty,-3] \cup [-2,2]$

$x^3+3x^2-4x-12 \le 0$

$x^2(x+3) - 4(x+3) \le 0$

$(x^2-4)(x+3) \le 0$

$(x+2)(x-2)(x+3) \le 0$

		-3		-2		2	
$x+3$	$-$		$+$		$+$		$+$
$x+2$	$-$		$-$		$+$		$+$
$x-2$	$-$		$-$		$-$		$+$
$f(x)$	$-$		$+$		$-$		$+$

$\Rightarrow (-\infty,-3] \cup [-2,2]$.

Q27.

Simplifying the expression: $\dfrac{x}{x^2+11x+30}-\dfrac{5}{x^2+9x+20}=$

A) $\dfrac{x-6}{x+5}$

B) $\dfrac{x+6}{x+4}$

C) $\dfrac{-1}{(x+6)(x+4)}$

D) $\dfrac{1}{x+4}$

E) $\dfrac{x-6}{(x+6)(x+4)}$

$$\frac{x}{(x+6)(x+5)}-\frac{5}{(x+4)(x+5)}=$$

$$\frac{x(x+4)-5(x+6)}{(x+6)(x+5)(x+4)}=\frac{x^2+4x-5x-30}{(x+6)(x+5)(x+4)}$$

$$=\frac{x^2-x-30}{(x+6)(x+5)(x+4)}=\frac{(x+5)(x-6)}{(x+6)(x+5)(x+4)}$$

$$=\frac{x-6}{(x+6)(x+4)}$$

Q28.

If $f(x)$ is a polynomial of degree 3 with real coefficients and having zeros: -3, 1, 4; and $f(2)=30$, then $f(x)=$

A) $-3x^3-2x^2-11x+12$

B) $x^3-2x^2-11x+12$

C) $-x^3+2x^2+11x-12$

D) $-3x^3+6x^2+33x-36$

E) $3x^3-6x^2-33x+36$

$$f(x)=a(x+3)(x-1)(x-4)$$

$$f(x)=a(x^3-2x^2-11x+12)$$

$$f(2)=a(8-8-22+12)=30$$

$$-10a=30$$

$$a=-3$$

$$\Rightarrow f(x)=-3x^3+6x^2+33x-36.$$

Q29.

Performing the division $\dfrac{2x^4-3x^2-3x+1}{x^2+x-1}$, the quotient $Q(x)$ and the remainder $R(x)$ are:

A) $Q(x)=2x^2-2x+1;\ R(x)=-6x+2$

B) $Q(x)=2x^2+2x+1;\ R(x)=6x+2$

C) $Q(x)=2x^2+2x-1;\ R(x)=6x-2$

D) $Q(x)=2x^2-2x-1;\ R(x)=-6x-2$

E) $Q(x)=2x^2-2x;\ R(x)=-6x$

$2x^2-2x+1$

$x^2+x-1 \;\big)\; 2x^4-3x^2-3x+1$

$2x^4+2x^3-2x^2$

$-2x^3-x^2-3$

$-2x^3-2x^2+$

x^2-5x

x^2+x

$-6x$

Q30.

Given that $-2i$ is a zero of the polynomial $p(x)=2x^4-x^3+7x^2-4x-4$ then the sum of the **real zeros** of $p(x)$ is:

A) $\frac{1}{2}$

B) 0

C) $-\frac{1}{2}$

D) $\frac{3}{2}$

E) $-\frac{3}{2}$

$2i \;|\; 2 \quad -1 \quad 7 \quad -4 \quad -4$

$\quad 4i \quad -8-2i \quad -2i+4 \quad -4i^2$

$2 \quad 4i-1 \quad -1-2i \quad -2i \quad 0$

$-2i \;|\; 2 \quad 4i-1 \quad -1-2i \quad -2i$

$\quad -4i \quad 2i \quad 2i$

$2 \quad -1 \quad -1 \quad 0$

$2x^2-x-1=0$

$(2x+1)(x-1) \Rightarrow x=-\frac{1}{2}$

$x=1$

Q31.

If $x, y > 0$, then $\left(\dfrac{x^{5/6}}{3y^{3/4}}\right)^2 \left(\dfrac{x^6}{8y^3}\right)^{-2/3} =$

A) $\dfrac{2}{9}\dfrac{y^{1/2}}{x^{7/3}}$

B) $\dfrac{4}{9}\dfrac{x^{17/3}}{y^{1/2}}$

C) $\dfrac{36y^{7/3}}{x^{7/3}}$

D) $\dfrac{2}{9}\dfrac{x^{1/3}}{y^{2/3}}$

E) $\dfrac{4}{9}\dfrac{y^{1/2}}{x^{7/3}}$

$$= \frac{x^{10/6}\, y^{-6/4}}{9}\; x^{-12/3}\, y^{2}\, 8^{2/3}$$

$$= \frac{4}{9}\, x^{10/6 - \frac{12}{3}}\, y^{-\frac{6}{4}+2}$$

$$= \frac{4}{9}\, x^{-14/6}\, y^{\frac{-6+8}{4}}$$

$$= \frac{4}{9}\, x^{-7/3}\, y^{2/4}$$

Q32.
A student has scores 64 in the first test and 78 in the second test of his Math 001 course. What score on the third test will give the student an average of 80 for the three tests? [All tests are out of 100]

A) 92
B) 81
C) 98
D) 85
E) 90

$$\frac{64+78+x}{3} = 80$$

$$x = 240 - 64 - 78$$

$$x = 98$$

Q33.

One of the factors of the expression $10(2y-1)^2 - 19(2y-1) - 15$ is:

A) $4y+7$
B) $5y-7$
C) $5y+1$
D) $4y+1$
E) $4y-7$

let $u = 2y-1$

$10u^2 - 19u - 15 = 0$

$(5u+3)(2u-5) = 0$

$(10y-2)(4y-7) = 0$

Q34.

If $f(x) = 5x^4 - 12x^2 + 2x + k$ is divided by $x-2$, the remainder is 28, then $k=$

A) -36
B) 16
C) -8
D) 8
E) -16

$f(2) = 5(16) - 12(4) + 4 + k = 2$

$= 80 - 48 + 4 + k = 28$

$\Rightarrow 36 + k = 28$

$k = -8$

Using Remainder Theorem

Q35.

The solution set of the equation $\left|x^2-2\right|-\left|2x\right|=0$ contains:

A) two positive irrational numbers and two nonreal complex numbers
B) two positive and two negative irrational numbers
C) two positive irrational numbers and two negative rational numbers
D) four nonreal complex numbers
E) two positive rational numbers and two negative irrational numbers

$|x^2-2| = |2x|$

$\Rightarrow$ $x^2-2 = 2x$ or $x^2-2 = -2x$

$x^2-2-2x=0$ or $x^2-2+2x=0$

$x = \dfrac{2 \pm \sqrt{12}}{2}$ or $x = \dfrac{-2 \pm \sqrt{12}}{2}$

$x = 1 \pm \sqrt{3}$ $\quad$ $x = -1 \pm \sqrt{3}$

Q36.

The solution set, in interval notation, of the inequality $\left|10-4x\right|+1 \geq 5$ is:

A) $\left[\frac{3}{2}, \infty\right)$

B) $\left(-\infty, -\frac{7}{2}\right] \cup \left[-\frac{3}{2}, \infty\right)$

C) $\left(-\infty, \frac{3}{2}\right] \cup \left[\frac{7}{2}, \infty\right)$

D) $\left(-\infty, \frac{7}{2}\right]$

E) $\left[\frac{3}{2}, \frac{7}{2}\right]$

$|10-4x| \geq 4$

$10-4x \geq 4$ or $10-4x \leq -4$

$4x-10 \leq -4$ or $4x-10 \geq 4$

$4x \leq 6$

$x \leq \frac{3}{2}$ or $4x \geq 14$

$x \geq \frac{7}{2}$